# Production scheduling of continuous make-andpack processes with byproducts recycling 

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#### Abstract

In this work, the scheduling problem of continuous make-and-pack industries is considered. A continuous precedence-based MILP model is proposed for multistage continuous processes, considering flexible intermediate storage. Extending previously proposed precedence-based MILP models, multiple campaigns of the same recipe can be stored simultaneously in a storage tank. Explicit resource constraints related to the generation and recycling of byproduct are introduced, to achieve a better utilization of the available resources. Several case studies, inspired by a large-scale consumer goods industry have been solved, to illustrate the applicability of the proposed frameworks. It is illustrated that good quality schedules are obtained in reasonable solution times.


Keywords: scheduling optimisation, MILP, make-and-pack, continuous processes

## 1. Introduction

Within the current climate of business globalization, modern process industries have to produce a plethora of final products that can address the needs and demand of multiple customers. Hence, scheduling optimization is becoming a vital process and decisionmakers tend to exploit recent advances in computer-aided optimization methods (Harjunkoski et al., 2014). Nowadays, several companies from various industrial sectors, such as food and beverages, pharmaceuticals, chemicals and fast-moving consumer goods (FMCGs), have adopted make-and-pack production processes. Due to variable production rates, a challenge typically met in continuous make-and-pack processes is the necessity to synchronize the production rates of consecutive stages (Klanke et al., 2020). Thus, continuous stages are often decoupled by deploying intermediate storage vessels (Méndez and Cerdá, 2002). Furthermore, product-dependent changeovers, mainly occurred by cleaning operations, have to be minimized to increase the productivity of production facilities. In cases when cleaning with water can affect the quality of products, an undesirable amount of byproduct waste is generated between two consecutive campaigns. Usually, the byproducts can be recycled into the next production campaigns. This industrial policy is typically met in liquid detergents industries. (Elekidis et al., 2019).

## 2. Problem statement

The scheduling problem under consideration has been inspired by a continuous make-and-pack process of a real-life, large-scale consumer goods industry (Elekidis, Corominas, and Georgiadis, 2019). Several intermediate products are produced through a continuous formulation stage (stage 1), while a plethora of final products is processed in the packing stage (stage 2), to satisfy multiple customer orders. Due to the different production rates of the two stages, a varying production bottleneck can be detected in
both stages depending on the specific product characteristics. To overcome this limitation and to achieve a better synchronization between the two stages, flexible intermediate buffer tanks are utilized. If an intermediate product is temporally stored in an intermediate vessel both stages operate at their maximum speed. Otherwise, the production rate is determined by the slowest stage. Furthermore, multiple changeovers take place between consecutive production campaigns due to the necessary cleaning operations. However, cleaning with water is not allowed, since water can affect the quality of the products, while the generation of an undesirable amount of foam also occurs. Under these circumstances, cleaning is implemented by generating an amount of byproduct waste. According to industrial policies, the waste can be temporally stored in tanks and it can then be recycled and utilized into the next intermediate products. The plant layout is also depicted in Figure 1.


Packing Stage

Intermediate Storage
Packing Line 1


Packing Line 2

Packing Line $N$

Figure 1 Plant layout

## 3. MILP model

A continuous-time, precedence-based, MILP model for continuous make-and-pack processes is proposed, considering intermediate storage tanks. The model consists of assignment, timing and sequencing constraints, similar to relevant frameworks (Elekidis et al. 2019). Mass balance constraints for the storage vessels are also imposed based on the framework of Méndez and Cerdá, (2002). Immediate precedence variables are used for sequencing operations in the processing units, while general precedence variables are utilized to define the sequence of storage operations. In comparison with discrete-time formulations, only a binary variable is introduced for handling the mass balance constraints. This auxiliary binary variable defines if the packing operation of a product order starts before or after the completion of its formulation process.
In the MILP model of Méndez and Cerdá (2002), it is assumed that each product can be stored in a vessel only if the packing operations of the previously stored products have been completed. However, this assumption is not in full agreement with industrial practice, since product orders produced by the same intermediate could potentially share the same storage tank. The proposed MILP model differs from the model of Méndez and Cerdá, (2002), since by integrating additional mass balance constraints and introducing a new set of auxiliary binary variables, multiple production orders, produced by the same

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 recyclingintermediate product type, are allowed to be stored in the same storage tank simultaneously.
The synchronisation of stages is achieved via proper timing constraints. An intermediate product can either be temporarily stored into a buffer tank or routed directly to a packing line, bypassing the storage vessels. Timing constraints guarantee that if a product bypasses the storage tank, the process of the two stages will start simultaneously.
Additionally, efficient resource-constraints, related to generation and recycling of byproduct waste are proposed to consider potential benefits by their utilization in the plant. The main objective is the minimization of the total cost, taking into account the costs of changeovers, the idle time, the processing time and the cost of the generated byproduct waste.

### 3.1. Mass balance constraints for product orders produced by the same intermediate product type (recipe)

Production campaigns which are made by the same intermediate product, can be stored simultaneously in the same buffer tank for some period of time. Due to the varying production rates, a production bottleneck can be posed in both stages depending on the specific product features. To prevent the overloading of storage vessels, mass balance constraints have to be enforced at two time points for each product. The first time point corresponds to the end of each formulation operation while the second one corresponds to the starting time of each packing operation (Méndez and Cerdá, 2002). Extending the work of Méndez and Cerdá (2002), a new set of auxiliary binary variables is introduced to accurately handle mass balance constraints. In case multiple intermediate campaigns are stored simultaneously at the same storage tanks, the proposed binary variables define if an operation (formulation or packing process) starts before or after the time point under consideration. The new binary variables are determined via a set of big-M constraints.
The total stored amount at a specific time point is defined as the total inserted amount minus the total amount exported from the buffer tank. The inserted and exported amounts are efficiently defined via a set of big-M constraints. It should be noted that the vast majority of scheduling approaches for make-and-pack processes rely on discrete-time formulations to handle material balances around storage equipment thus, resulting in computationally intractable models.

### 3.2. Modelling of byproducts

In scheduling problem under consideration, a set of product orders $i \in I$ is allocated to a set of production units $j \in J$, at each production stage $s \in S$. A subset of units $j$ is able to process product order $i,\left(j \in J I_{i}\right)$ at stage $s\left(j \in J S_{s}\right)$. Additionally, a subset of processing units $j \in J$ is able to process product orders $i$, given by subset $I J_{j}$. The maximum production rate of each product $i$, at stage $s$, is given by the parameter $r_{i, s}$.
The vast majority of product-dependent changeovers, usually take place among the production of consecutive production campaigns, $i$ and $i^{\prime}$, are related to cleaning operations. In liquid detergents industries, a significant amount of waste is generated during the changeover time, $n_{i, i^{\prime}}$, which can be recycled into the next production campaigns if the quality of products is not affected. Each processing unit $j$ of formulation stage $\left(j \in J S_{s=1}\right)$, is connected with a dedicated storage vessel, in which the generated waste can be temporarily stored. Since the capacity of byproduct vessels have to be fully satisfied, a set of mass balance constraints are introduced, without utilising further binary variables or considering any discretisation of time.

$$
\begin{align*}
& O_{i}=L I_{i}+\sum_{i^{\prime} \in I: i^{\prime} \neq i} \sum_{j \in\left(J S_{1} \cap J I_{i} \cap J I_{i^{\prime}}\right)} X_{i, i^{\prime}, j} n_{i, i^{\prime}} r_{i^{\prime}, 1}-W_{i} \forall i \in I  \tag{1}\\
& L I_{i} \geq O_{i^{\prime}}-d m_{i}\left(1-\sum_{j \in\left(J S_{1} \cap J I_{i} \cap J I_{i^{\prime}}\right)} X_{i^{\prime}, i, j}\right) \forall i \in I, i^{\prime} \in I: i^{\prime} \neq i  \tag{2}\\
& L I_{i} \leq O_{i^{\prime}}+d m_{i}\left(1-\sum_{j \in\left(J S_{1} \cap J I_{i} \cap J I_{i^{\prime}}\right)} X_{i^{\prime}, i, j}\right) \forall i \in I, i^{\prime} \in I: i^{\prime} \neq i  \tag{3}\\
& L I_{i} \leq i w_{j} Y_{i, j}+d m_{i} \sum_{i^{\prime} \in I J_{j}: i^{\prime} \neq i} X_{i, i^{\prime}, j} \forall i \in I, j \in\left(J S_{1} \cap J I_{i}\right)  \tag{4}\\
& L I_{i} \geq i w_{j} Y_{i, j}-d m_{i} \sum_{i^{\prime} \in I J_{j}: i^{\prime} \neq i} X_{i, i^{\prime}, j} \forall i \in I, j \in\left(J S_{1} \cap J I_{i}\right)  \tag{5}\\
& W_{i} \leq d m_{i} a a_{i} \quad \forall i \in I I_{i} \forall  \tag{6}\\
& W_{i} \leq L_{i} \quad \forall i \in I \quad  \tag{7}\\
& O_{i} \leq c p_{j} Y_{i, j} \quad \forall i \in I, j \in\left(J S_{1} \cap J I_{i}\right)  \tag{8}\\
& R W_{j} \geq O_{i}-c p_{j}\left(1-Y_{i, j}\right)-c p_{j} \sum_{i^{\prime} \in I J_{j}: i^{\prime} \neq i} X_{i, i^{\prime}, j} \forall i \in I, j \in\left(J S_{1} \cap J I_{i}\right) \tag{9}
\end{align*}
$$

The accumulated amount of byproduct $O_{i}$ is calculated at the end of each changeover. According to mass balance constraints (1), the variable $O_{i}$ is equal to the previously accumulated byproduct amount $L I_{i}$, plus the waste generated by the cleaning operations that take place ( $n_{i, i^{\prime}} r_{i^{\prime}, 1}$ ), minus the recycled amount during the production of product $i$, $W_{i}$. In particular, it is assumed that during the changeover time $n_{i, i^{\prime}}$, between two consecutive campaigns $i$ and $i^{\prime}$, the production rate of byproduct is equal to the maximum production rate of campaign $i^{\prime}\left(r_{i^{\prime}, 1}\right)$. According to constraints (2) and (3), if product $i^{\prime}$ is produced immediately before product $i,\left(X_{i^{\prime}, i, j}=1\right)$, the accumulated byproduct is equal to variable $O_{i}$. Constraints (4) and (5) ensure that the accumulated amount of byproduct at the beginning of the first campaign $\left(X_{i^{\prime}, i, j}=0\right)$, is either equal to zero or equal to the initial byproduct $\left(i w_{j}\right)$ only if the product $i$ is allocated at unit $j\left(Y_{i, j}=1\right)$. The demand parameter, $d m_{i}$ plays the role of the big-M value in constraints (4) and (5). The maximum percentage that can be recycled is expressed by the parameter $a_{i}$. Constraints (8) ensures that the stored amount of byproduct, will not surpass the related capacity $\left(c p_{j}\right)$ of each tank. The remained amount of waste $R W_{j}$ at the end of the time horizon, is equal to the generated waste of the last production campaign $O_{i}$ of each unit.

## 4. Application studies

In order to evaluate the efficiency and applicability of the proposed MILP model, 4 case studies are considered. They include 3 formulation lines, 3 packing lines and two

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intermediate storage tanks. Due to the large number of final products, the decomposition algorithm of Elekidis et al. (2019), is utilized to solve the large-scale MILP model. To assess the benefits of the intermediate storage tanks, two case studies have been solved, with (decoupled layout) and without (coupled layout) the utilization of intermediate storage tanks. The minimization of total operational time is the main objective. The results are summarized in Table 1. It is illustrated that the utilization of flexible intermediate buffer tanks leads to a better synchronization of the production stages resulting in increased productivity. In particular, depending on the case study, the productivity gain ranges within $4.43 \%$ (case 2 ) and $17.29 \%$ (case 1 ).
Furthermore, the case studies have been also solved considering the minimization of total cost. Near-optimal solutions are obtained within a total CPU time of 3600s, while a zerooptimality gap is achieved at each iteration. Results are summarized in Table 2. The individual costs represent relative monetary units (rmu) and they are also presented in Table 2. It is observed that the biggest percentage of the total cost, reflects the idle time as it valued by $30 \mathrm{rmu} / \mathrm{h}$. Although the percentage of the changeover cost is gradually decreased in larger cases, from $17 \%$ (case 1) to $8 \%$ (case 4), the percentage of idle time cost is steadily increased, from $47 \%$ (case 1) to $77 \%$ (case 4).


Figure 2 Total produced and recycled amount in each case

Table 1 Comparison of the total operational time of the two layouts

| Products | Coupled <br> layout $^{*}$ | Decoupled <br> layout* $^{*}$ | Difference <br> (hours)* | Productivity gain <br> (\%) |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Case 1 | 20 | 186.63 | 225.63 | 39.00 | $17.29 \%$ |
| Case 2 | 50 | 355.34 | 371.82 | 16.48 | $4.43 \%$ |
| Case 3 | 70 | 422.14 | 442.70 | 20.55 | $4.64 \%$ |
| Case 4 | 100 | 596.23 | 680.88 | 84.64 | $12.43 \%$ |
| * |  |  |  |  |  |

*the values represent the total operational time of all production units of both stages in hours

Table 2 Results for larger problem instances - Cost distribution

|  | Products | TC* | COC* | ITC* | PTC* | WC* |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Case 1 | 20 | 1301 | $221(17 \%)$ | $614(47 \%)$ | $385(30 \%)$ | $79(6 \%)$ |
| Case 2 | 50 | 1999 | $231(12 \%)$ | $1224(61 \%)$ | $470(24 \%)$ | $72(4 \%)$ |
| Case 3 | 70 | 3505 | $299(9 \%)$ | $2592(74 \%)$ | $541(15 \%)$ | $72(2 \%)$ |
| Case 4 | 100 | 5455 | $422(8 \%)$ | $4193(77 \%)$ | $757(14 \%)$ | $82(2 \%)$ |

${ }^{*} T C=$ Total cost, $C O C=$ Changeover cost, ITC=idle time cost, $P T C=$ processing time cost, WC=waste cost
${ }^{* *}$ COC $=10 \mathrm{rmu} / \mathrm{h}, \mathrm{ITC}=30 \mathrm{rmu} / \mathrm{h}, \mathrm{PTC}=1 \mathrm{rmu} / \mathrm{h}, \mathrm{WC}=0.5 \mathrm{rmu} / \mathrm{kg}$
Figure 2 shows, the total produced amount and the amount of recycles as well, for each case. It is illustrated that the byproduct recycles constitute a significant percentage of the total produced amount, which ranges from $6.1 \%$ (Case 3) to $7.57 \%$ (Case 4). Hence, it is evident that the utilization of this recycling policy and the consideration of the proposed byproduct constraints lead to better utilization of raw materials and a significant reduction of material cost.

## 5. Conclusions

In this work a precedence-based MILP model, for the scheduling of continuous, make-and-pack industries is presented. The utilization of intermediate buffers can provide a better synchronization between the two production stages. The integration of byproduct recycling constraints leads to a better use of the available raw materials and to a significant reduction of waste. The proposed MILP model can constitute the basis of an optimization tool to assist decision-makers with rigorous scheduling solutions under a dynamic environment.

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