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Comparison of micromechanical elasticity models for weakly cemented granular materials

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ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ ΙΚΥ

ΕΘΝ. ΜΕΤΣΟΒ. ΠΟΛΥΤΕΧΝΕΙΟΝ · ΑΠΛΕ · ΠΡΟΜΗΘΕΥΣ ΠΥΡΦΟΡΟΣ

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for cemented granular materials (contact stiffness)

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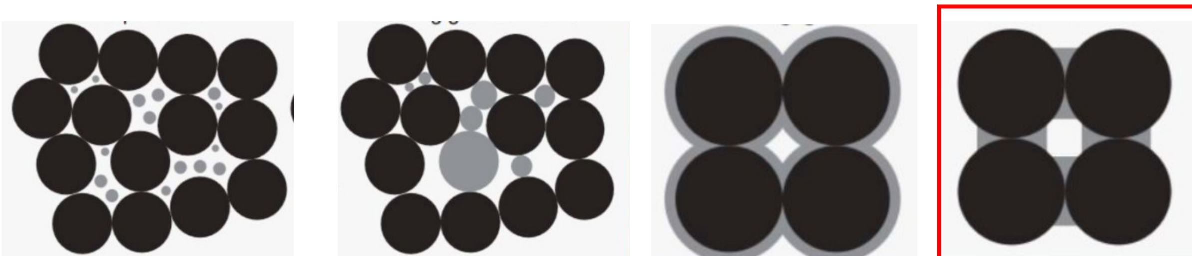
Main interaction mechanisms of grains & cement materials



Pore filling | Load bearing | Grain coating | Cementation (Bonded grains)

(after Waite et al., 2004)

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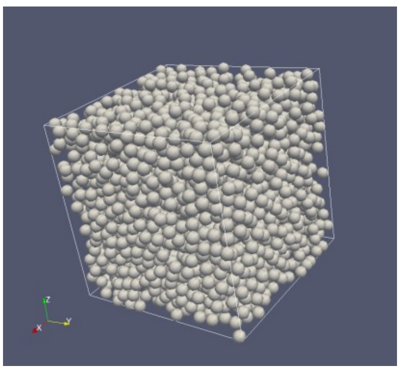
Discrete numerical simulations
Discrete Element Method (DEM)

DEM is a sophisticated but simple method
to numerically model granular materials at their particle scale.



It uses:
- Particles (spheres)
- Newton's second law
- Contact model laws
- Numerical schemes

Today's DEM are primarily for beads/sands
Clay-type DEM are on the way.

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Contact models for cemented materials

1. Potyondy & Cundall 2004

2. Brown, Chen & Ooi 2014

3. Langlois 2015 (Theocharis, Roux & Langlois, 2020)

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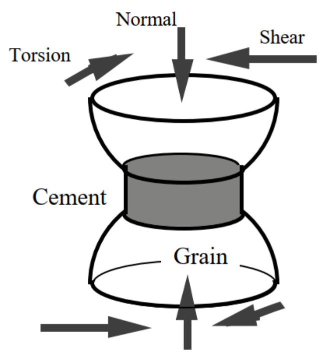
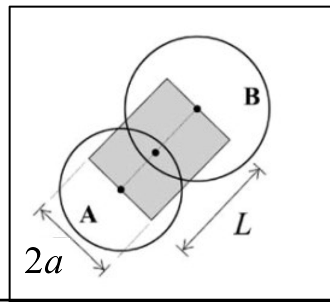
Contact Laws for cemented materials - 1



Potyondy & Cundall 2004

$$\Delta F_n = k_{ng}\Delta U_n + k_{nc}A_c\Delta U_n = \left(k_{ng} + k_{nc}A_c\right)\Delta U_n$$

$$k_n = k_{ng} + k_{nc}A_c = 4\mathrm{R}\,E_g + \frac{E_c}{L}\pi a^2$$



$$E_g = 70GPa, v = 0.3$$

$$E_c = 9.23GPa, v = 0.3$$

$$k_n = 140 + 29\frac{a^2}{L}$$

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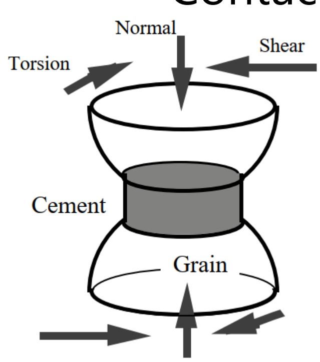
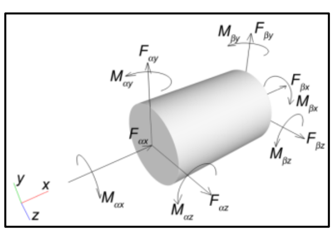
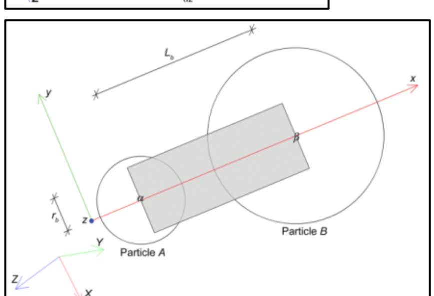
Contact Laws for cemented materials - 2



Brown, Chen & Ooi 2014





$$\Delta F_n = k_{nc}\Delta U_n$$

$$k_n = k_{nc} = \frac{E_c}{L}\pi a^2$$

$$E_g = 70GPa, v = 0.3$$

$$E_c = 9.23GPa, v = 0.3$$

$$k_n = 29\frac{a^2}{L}$$

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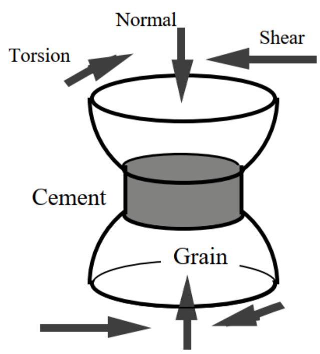
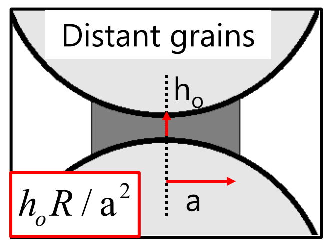
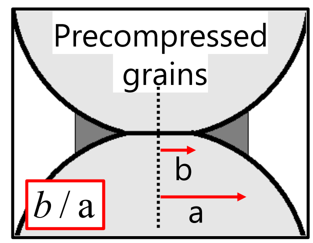
Contact Laws for cemented materials - 3



$$\Lambda_n = (2/\pi)(G_c / G_g)[(1-\nu_g)(1-\nu_c)/(1-2\nu_c)]$$

$$\alpha = a / R \qquad \rightarrow \alpha / \Lambda_n$$





- Dvorkin et al. 1991
- Dvorkin et al. 1994
- Langlois 2015
- Theocharis et al. 2020

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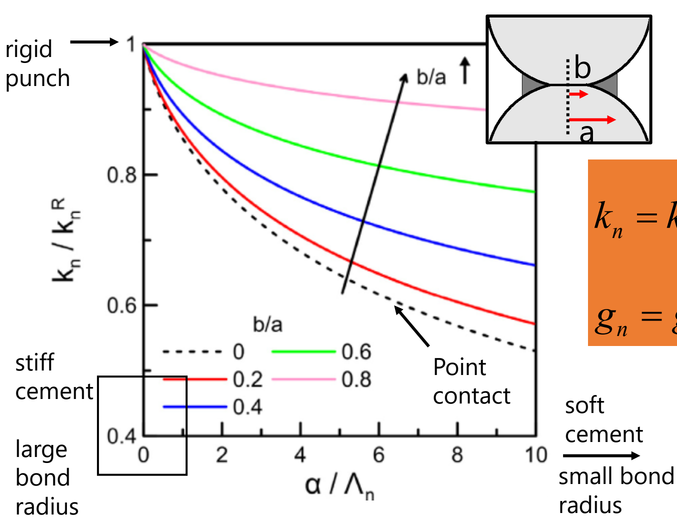
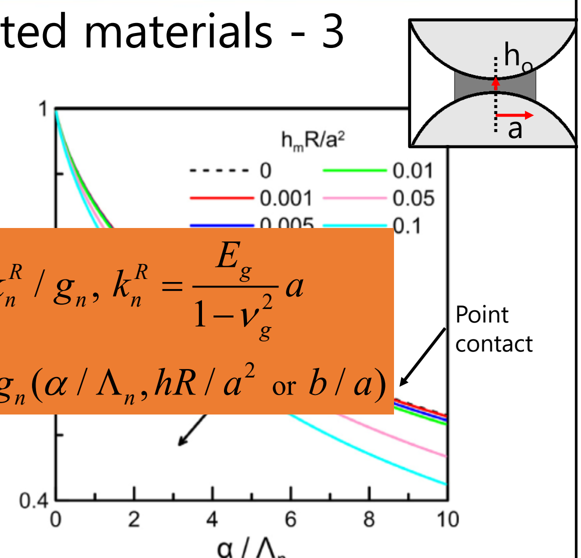
Contact Laws for cemented materials - 3

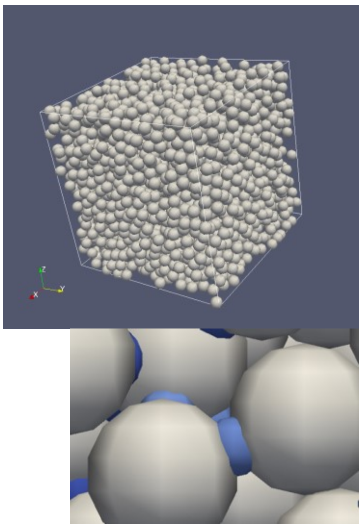




$$k_n = k_n^R / g_n, \; k_n^R = \frac{E_g}{1-\nu_g^2} a$$

$$g_n = g_n(\alpha / \Lambda_n, hR/a^2 \text{ or } b/a)$$

DEM Samples



Full periodic cell
Monodisperse distribution, R=0.5
4000 grains
Several initial conditions

No	Pressure (kPa)	Porosity	Coordination number	Rattlers
1	80	0.423 ± 0.002	4.75 ± 0.027	296 ± 53 (7.4%)
2	100	0.406 ± 0.001	4.79 ± 0.023	344 ± 18 (8.6%)
3	100	0.365 ± 0.0006	4.82 ± 0.011	494 ± 76 (12.3%)
4	100	0.373 ± 0.0008	5.86 ± 0.008	55 ± 14 (1.4%)
5	100	0.362 ± 0.001	6.17 ± 0.004	45 ± 4 (1.1%)

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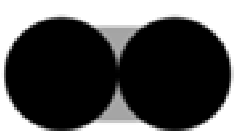
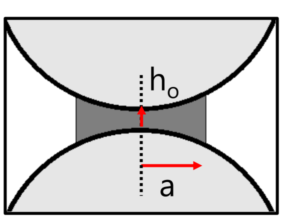
Cement saturation (*S*)

- Cement saturation (*S*) is defined as the cement volume over the initial void volume.
- The cement is homogenously created over all grains.





2 parameters are needed to define cement saturation:

maximum cement distance: h_o = 0 - 0.15
cement radius: a = 0.01 - 0.3 (R=0.5)

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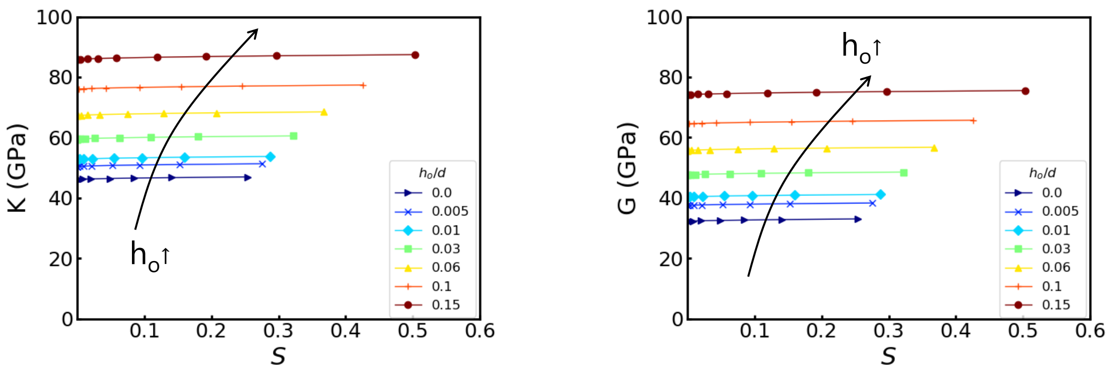
Elastic Moduli with cement saturation – 1 (Potyondy & Cundall, 2004)
Dense sample No 4 – Qualitatively similar results for all samples



- Increase of moduli with cement distance
- Practically constant moduli with cement radius and cement saturation
- Different results for same cement saturation but different cement distribution 13

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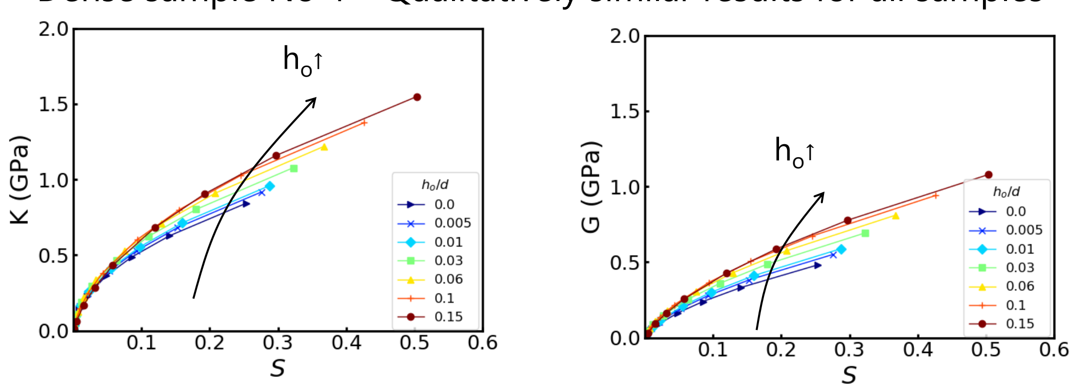
Elastic Moduli with cement saturation – 2 (Brown, Chen & Ooi 2014)
Dense sample No 4 – Qualitatively similar results for all samples



- Increase of moduli with cement distance, cement radius and cement saturation
- Different results for same cement saturation but different cement distribution 14
- Increase with cement distance more important than with cement radius

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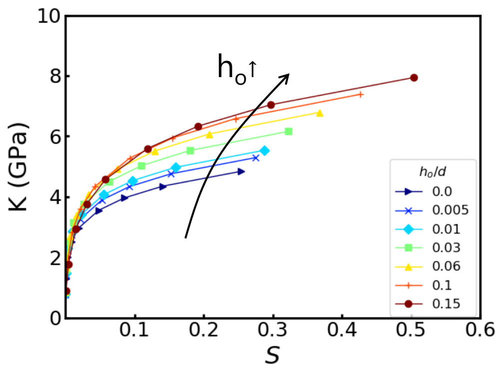
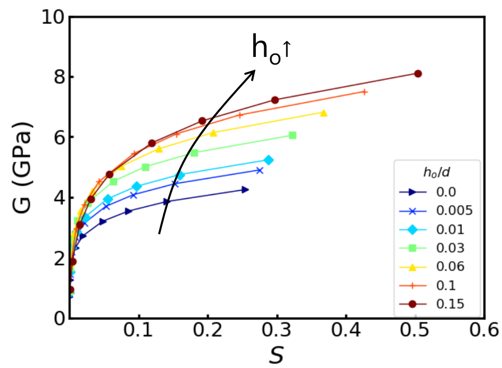
Elastic Moduli with cement saturation – 3 (Theocharis et al. 2020)
Dense sample No 4 – Qualitatively similar results for all samples





- Increase of moduli with cement distance, cement radius and cement saturation
- Different results for same cement saturation but different cement distribution
- Increase with cement distance more important than with cement radius

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Conclusions

- Contact stiffness for cemented materials based on three contact laws:
  - (1) Potyondy & Cundall 2004, Beam element - general assumptions
  - (2) Brown, Chen & Ooi, 2014, Timoshenko beam element
  - (3) Theocharis, Roux & Langlois, 2020, Elastic solution + Langlois formulation
- Elasticity parameters based on initial configurations & creation of cement
  - Different results for same cement saturation but different cement distribution

  For (1)
  - Practically constant moduli with cement radius and cement saturation
  - Increase of moduli with cement distance

  For (2) and (3)
  - Increase of moduli with cement distance, cement radius and cement saturation
- (1) Provides the most stiff contacts and highest elastic moduli
- (2) provides the lowest elastic moduli

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Thank you !

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