MedGU Mediterranean Geosciences Union
Annual Meeting | Istanbul, 25–28 November 2021
Springer | Publishing Partner

# Comparison of micromechanical elasticity models for cemented soils

Alexandros I. Theocharis[1], Achilleas G. Papadimitriou[1]

[1] National Technical University of Athens, School of Civil Engineering, Department of Geotechnical Engineering, Athens, Greece

`atheomec@mail.ntua.gr`

**Abstract.** Cemented soils have particles held together by a binding agent. Of particular interest are weakly cemented soils found in an intermediate state, between granular and solid, when loading begins to break their bonds. To properly understand and describe such materials, appropriate contact laws are needed. This constitutive behavior has two main parts, elasticity and bond rupture; several works have focused on the bonds' failure criterion, and fewer have dealt with their elastic response. This work compares granular specimens' macroscopic elastic behavior based on various contact bond laws employed with the Discrete Element Method (DEM). An advanced bond law proposed and evaluated by one of the authors is compared with other two simpler laws in terms of formulation but well-established in the literature. Cementation is applied to DEM samples with spherical grains in different isotropic states with different densities and contact connectivities. Overall, all laws can capture some fundamental aspects of the samples' elasticity and its dependence on the bond characteristics (bond geometry, bond stiffness). However, as the bond law becomes more complex, more effects affect the macroscopic elastic response.

**Keywords:** Geomechanics, Cemented soils, Micro to macro response, Discrete Element Method

## 1 Introduction

Soils are granular materials with networks of grains and contacts. Different processes might eventually transform the contacts leading to the formation of cemented soils. Once cemented, grain assemblies behave like solids until bonds are broken. Sone examples of such materials are sand grains that turn into sandstones; in engineering practice, one may exploit the micro-organisms' cementation capacity to improve soil properties; a similar result may occur as gas hydrates form inside submarine sediments. Of particular interest are weakly cemented granular materials found in an intermediate state when loading breaks their bonds. To properly understand and describe such materials, appropriate contact laws are needed. Several works have focused on the bonds' failure criterion, and fewer have dealt with their elastic response. This work compares granular specimens' macroscopic elastic behavior based on three contact bond laws.

## 2 Contact laws and macroscopic elasticity

The bond laws are presented in the following in order of increasing complexity, at least regarding their final formulation. Focus is here given only on the normal (axial) bond stiffness being the most critical component related to the macroscopic elasticity of granular samples. However, the other parts of the bond laws (tangential, rolling, and pivoting stiffness) were also used in the calculations of this study but are not analyzed.

Brown et al. [1] presented a bond law, named herein Law1, based on the assumption of the Timoshenko beam theory, which considered axial, shear, and bending behavior of the bond. A cylindrical beam is considered joining the centers of two distant grains, and the response of the beam defines the behavior of the bond. Then the normal stiffness yields:

$$k_n^1 = \frac{E_c}{L} \pi a^2 \tag{1}$$

where $E_c$ is the Young modulus of the beam, L is the distance between the grain centers, and $\alpha$ is the beam's radius. Based on the Timoshenko beam theory, all additional stiffnesses are accordingly calculated.

Potyondy and Cundall [2] presented a bond law, named herein Law2, for rock; the primary assumption is the "parallel bond" concept, i.e. a bond similar to a cylindrical beam element that works in parallel to the contact of two particles. Therefore, the normal stiffness has two parts, one from the bond and one from the grain contact:

$$k_n^2 = 4\,\mathrm{R}\,E_g + \frac{E_c}{L} \pi a^2 \tag{2}$$

where $E_g$ is the grain's Young modulus and R is the mean radius of the grains in contact.

Finally, Theocharis et al. [3] presented a contact law based on the solution of the boundary value problem that includes two grains in distance or in contact bonded by an elastic cylindrical bond. Because the result could not be analytically represented, the normal stiffness was expressed through a correction function to a critical limit, the rigid punch. This limit is when the grains are rigid, and only the bond contributes to the elastic behavior. The rigid punch normal stiffness and the normal stiffness are then:

$$k_n^3 = k_n^R / g_n ;\; k_n^R = \frac{E_g}{1-\nu_g^2} a \tag{3}$$

where $g_n$ is the correction function that depends on the grains' distance and the elastic relation between the grains the cement.

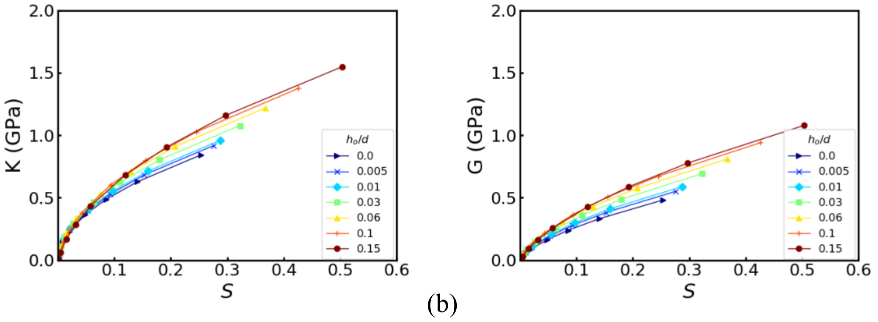
Cementation was applied to DEM samples with spherical grains in different isotropic states with different densities and contact connectivities. **Table 1** presents the employed samples, while the cement was assumed to be applied as a cylinder of predefined radius to all grains in contact or in distance with a distance less than a predefined limit.

The macroscopic elasticity is presented in terms of bulk and shear moduli. Due to the cementation procedure, two parameters are critical for the definition of the elasticity: the cement volume over the initial void volume (S, cement saturation) and the limit distance ($h_o$) - over the grain diameter (d) - below which two grains are cemented.

**Table 1.** DEM samples used for comparison [3]

| No | Pressure (kPa) | Porosity | Coordination number | Rattlers |
|---|---|---|---|---|
| 1 | 80 | 0.423 ± 0.002 | 4.75 ± 0.027 | 296 ± 53 (7.4%) |
| 2 | 100 | 0.406 ± 0.001 | 4.79 ± 0.023 | 344 ± 18 (8.6%) |
| 3 | 100 | 0.365 ± 0.0006 | 4.82 ± 0.011 | 494 ± 76 (12.3%) |
| 4 | 100 | 0.373 ± 0.0008 | 5.86 ± 0.008 | 55 ± 14 (1.4%) |
| 5 | 100 | 0.362 ± 0.001 | 6.17 ± 0.004 | 45 ± 4 (1.1%) |

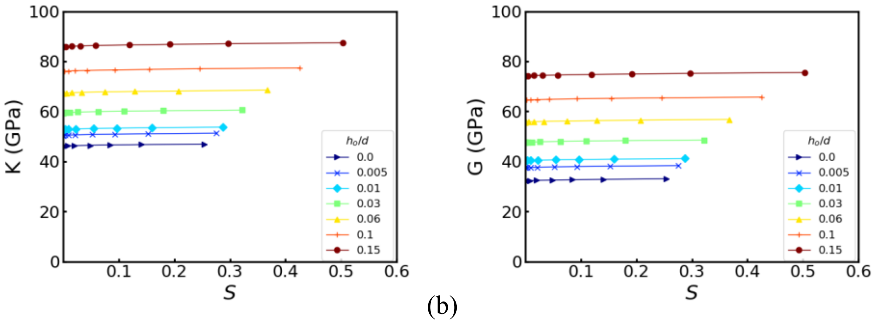
Based on these two parameters, the macroscopic moduli of one indicative DEM sample (Sample No 4) are presented for the three contact laws in **Fig. 1**, **Fig. 2**, and **Fig. 3**.



(a) (b)

**Fig. 1.** Elastic Moduli with cement saturation for dense sample No 4 and Law1 [1]

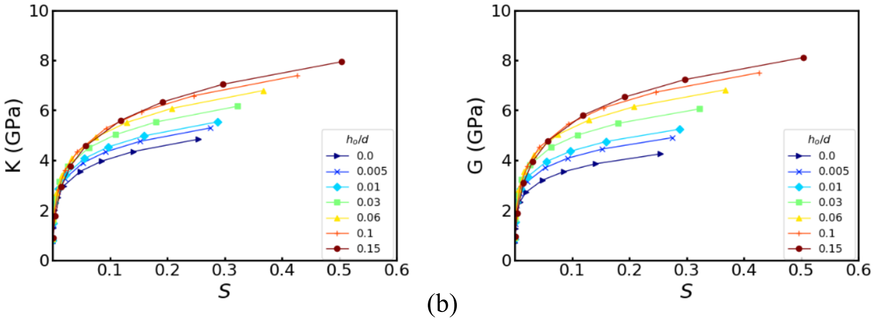
A different order of magnitude is presented for the different laws; Law1 proposes moduli up to 1.5Gpa, Law3 up to 8GPa, and Law2 up to 75GPa. Additionally, when saturation tends to zero, three different situations are observed. Law1 tends to zero as Eq. (1) provides stiffness only when bonds exist, Law2 tends to a very high value due to the contribution of the grain part in Eq. (2), Law3 tends to a non-zero value, lower than for Law2.



(a) (b)

**Fig. 2.** Elastic Moduli with cement saturation for dense sample No 4 and Law2 [2]

In all cases, different results arise for the same cement saturation but different cement distribution. Furthermore, for Law1 and Law3, moduli increase with cement distance, cement radius, and cement saturation. In contrast, for Law2, due to the very high contribution of the grain part in Eq. (2), practically constant moduli are observed with cement radius and cement saturation. Nevertheless, even in this case, there is an increase with cement distance. Overall, Law2 provides the stiffest contacts and the highest elastic moduli, while Law1 provides the lowest elastic moduli.



(a) (b)

**Fig. 3.** Elastic Moduli with cement saturation for dense sample No 4 and Law3 [3]

## 3 Conclusions

This work compared three contact laws in terms of their effect on the macroscopic elastic response of various DEM samples. The comparison shows that all laws can capture some fundamental aspects of the samples' elastic moduli and their dependence on the bond characteristics (bond geometry, bond stiffness). However, as the law becomes more complex, more effects are added, affecting the macroscopic elastic response.

## 4 Acknowledgments

This work was co-financed by Greece and the European Union (European Social Fund) through the Operational Program "Human Resources Development, Education and Lifelong Learning", in the framework of the Act "Support of postgraduate researchers - B cycle" (MIS 5033021) implemented by the State Scholarships Foundation (IKY).

## 5 References

1. Brown, N.J., J.-F. Chen, and J.Y. Ooi, *A bond model for DEM simulation of cementitious materials and deformable structures.* Granular Matter, 2014. 16(3): p. 299-311.
2. Potyondy, D.O. and P.A. Cundall, *A bonded-particle model for rock.* International Journal of Rock Mechanics and Mining Sciences, 2004. 41(8): p. 1329-1364.
3. Theocharis, A., J.-N. Roux, and V. Langlois, *Elasticity of model weakly cemented granular materials: A numerical study.* International Journal of Solids and Structures, 2020. 193-194: p. 13-27.