Taxes, Social Insurance Contributions and Undeclared Labour

in Unionized Oligopoly

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Abstract

Undeclared Labour (UDL) constitutes a complex and multidimensional phenomenon that influences both the economy and the society of a state. Up to our days, state's policy focuses mostly on monitoring the labour market and imposing fines in order to tackle the UDL phenomenon. In May 2016, EE launched a European Platform to enhance cooperation in tackling undeclared work, setting three main aims: to encourage closer co-operation between Member States; to improve the capacity of different relevant authorities and actors to tackle undeclared work; and to increase awareness of issues relating to undeclared work. Furthermore, UDL has not yet been analyzed within I/O framework. In this paper we turn our attention to reducing the economic attractiveness of the UDL phenomenon, using a proper analytic theoretical model. In a unionized duopoly under decentralized wage bargaining context and proportional taxation, we suggest that there exists a trade-off between the imposed tax rate and the contributions for social insurance. The relation between tax and social contribution rates affects and determines the final amount of UDL in labour market, and thus it may be used as a supplementary policy tool to tackle UDL. Moreover, we investigate the way that UDL affects the other micro and macro sizes, using I/O analytical framework.

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1. Introduction

Undeclared work is defined as "any paid activities that are lawful as regards their nature but not declared to public authorities". It is a complex phenomenon associated with breaches of workers' rights, unfair competition, tax evasion and social security fraud. It affects governments, businesses and workers and it concerns various types of activities, ranging from informal household services to clandestine work by illegal residents, but excludes criminal activities.

The motivation to apply undeclared labor lies to the potential gain in avoiding taxes and social security contributions, social rights and the cost of complying with regulations.

From a macroeconomic point of view, undeclared labour reduces tax revenues (since employees declare no income and then no taxes are imputed) and undermines the financing of social security systems. To the extent that undeclared work competes with and even crowds out activities that comply with regulations, it is the main source of social dumping. In the case of undeclared work performed by individuals who are receiving benefits compensating their inactivity, there is also a dimension of social fraud.

From a microeconomic perspective, undeclared labour distorts fair competition among firms and causes productive inefficiencies, as informal businesses typically avoid access to formal services and inputs (e.g. credit) and prefer to stay small.

Undeclared labour is a decomposite phenomenon, that is influenced by a great range of economic, social, structural and cultural factors, tending to comprise a constraint to economic, fiscal, and social policies applied for the economic growth of an economy.

The fact that undeclared labour on one hand cannot be observed and on the other hand may be otherwise defined among countries, makes it even more difficult to establish credible evaluations about the growth of this phenomenon. However, a research, conducted on behalf of European Committee at 2004, while it accented important differences among countries regarding the qualitative characteristics as well as the size of undeclared labour, estimated undeclared labour's maximum values at 20% at some countries of Eastern and South Europe. Furthermore, a

Eurobarometer Survey, carried out in 2013, showed that in the EU 11% of Europeans admitted that they have bought goods or services involving undeclared work in the previous year, 4% concede that they themselves have received undeclared pay in return for work, while one in 30 (3%) has been paid partly in cash by his or her employer ("envelope wages").

Given the complexity and the heterogeneity of the phenomenon, there is no simple solution to confront it. Nevertheless, the resolution of the European Union's Council of 29 October 2003 on transforming undeclared work into regular employment proposed the following policies:

- Reducing the financial attractiveness of undeclared work stemming from the design of tax and benefit systems, and the permissiveness of the social protection system with regard to the performing of undeclared work;
- Administrative reform and simplification, with a view to reducing the cost of compliance with regulations;
- Strengthening the surveillance and sanction mechanisms, with the involvement of labour inspectorates, tax offices and social partners;
- Trans-national cooperation between Member States, and
- Awareness raising activities.

Regarding the first policy group of meters, European Committee concluded that there is still a great deal of actions to be done in order to balance both the motives and the disincentives offered by the social security systems. In particular, proposed policies concern the reservation of adequate income levels (taking into account the relation between benefits and contributions), the enforcement of exercising control over the labour market and over the persons entitled to social benefits and the imposition of proper economic penalties for tax and contribution evasion.

To gain all the above, policies should emphasize in:

I. Proper taxation of overtime work;
II. Maintaining the institutional minimum wages;
III. Regulating tax distortions between tax systems applied in wage earners and those applied to self-employed;
IV. Reducing the taxation of low productivity activities.

Even though during the past decades a broad range of methods has been developed to analyze the undeclared labour phenomenon, to understand its dimensions and causes, to formulate an appropriate policy to constrain its spread, neither this phenomenon has been examined with any available method, nor the discussion about which methodology is the most appropriate has still not come to an end. In particular, there has been an extended use of econometrics and applied statistics in the relevant researches. Surveys from international organizations (such as OECD, ILO, EU etc) based mostly on evidence and results of state audits also consist a notable framework. However, undeclared labour has not yet been approached or analyzed using the framework of industrial organization and game theoretic analytical toolkit.

With this research, we aspire to deliver a different approach, using the industrial's organization framework. Moreover, one of the main goals of this work is to propose a different policy for restraining the phenomenon of undeclared labour. As it is shown, the use of proper tax rates relative to those of social insurance could – under certain circumstances – restrain the economic attractiveness of this phenomenon.

The present analysis is organized as follows:

- In Section 2, we consider a – rather innovative with quite strong results – model with exogenously determined wages, where two firms are competing a la Cournot, the first firm declares its workers while the second one does not. The research focuses on the determination of the circumstances under which the second firm switches in worse economic position than the first one.
- In Section 3, we endogenize firms' choice of optimal percentage of undeclared labour, decentralized wage determination, inducting unionized oligopoly, under a proportional tax system both for firms and workers.

Finally, we summarize our major results and propose directions for further research at the Conclusions.

2. Exogenous Wage and Undeclared Labour in Oligopoly

2.1. The Model

Consider a homogeneous good sector where two firms, f_1 and f_2, compete by adjusting their quantities. We also assume a production function $q_i=L_i$ for both firms (q_i: the production of *i* firm, L_i: the workers used in *i* firm to produce q_i, *i*: 1, 2). The first firm insures its personnel and faces $(1+k)\cdot w$ unit labour cost[1], including contributions for social insurance, where *w* stands for wage and *k* for the percentage of the wage for social insurance contributions. The second firm decides not to insure its personnel and faces *w* unit labour cost (just the wage).

Additionally, both firms pay taxes of rate t on their declared net profits. Notice that, since the first firm declares and insures its workers, the whole payroll costs (meaning both wages and contributions for social security) should decrease the final net profits; while the second firm doesn't have this option, since undeclared labour cannot be shown at any public authority, including tax office. The tax functions form are as follows[2]:

f_1 profit's taxation $= t\cdot\left((p-(1+k)\cdot w)\cdot q_1\right)$

f_2 profit's taxation $= t\cdot(p\cdot q_2)$

Therefore, the first firm will pay contributions for social insurance and thus fewer taxes (since declared profits will be fewer), while the second firm will pay nothing for social security but more taxes (since declared profits will be significantly higher). It is clearly shown that there is an opportunity cost for firms, between taxation and contributions for social insurance.

Notice that at this stage of our early analysis, any choice of the firms to declare their workers or not, as well as the wage determination, are both considered exogenously. We assume that one firm acts in reverse to the other and examine which one is finally in better position. On the other hand, the wages are considered to be institutionally announced and apply for all firms in the economy (i.e., $w_1=w_2=w$). Our analysis does not, also, include any governmental surveillance or

[1] We normalize production per unit cost to zero.

[2] Quantities, profits and taxation for each firm must be a positive argument. Thus, in order our model to have internal solutions, we set $t < t_{cr} = \frac{1+(-1+k)w}{1+w+kw}$ and 0 < w < 0.5 .

compliance penalties. We simply examine the equilibrium of the market, when it is auto-regulated, without any further interventions.

2.2. Solving the Model

Let for tractability the reverse demand function be normalized to $P(Q) = 1 - Q$, where $Q = q_1 + q_2$. Then, given our setup, the firms' profit functions are as follows:

$$\Pi_1 = \left[P(Q) - (1+k)\cdot w\right]\cdot q_1 - t\cdot\left[\left(P(Q) - (1+k)\cdot w\right)\cdot q_1\right] \quad (1)$$

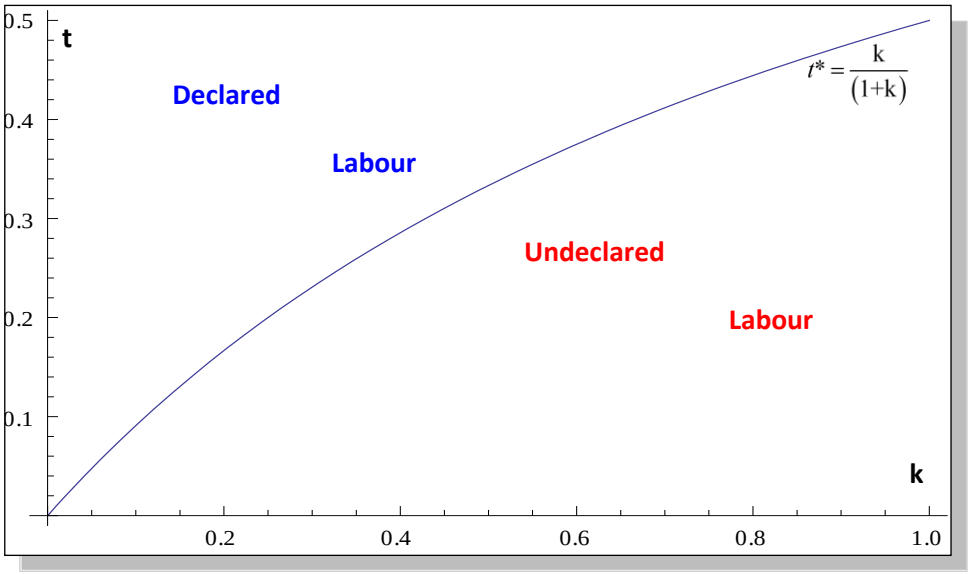
$$\Pi_2 = \left[P(Q) - w\right]\cdot q_2 - t\cdot\left[\left(P(Q) - w\right)\cdot q_2\right] \quad (2)$$

Taking the first order conditions and solving the model, we conclude that the quantities of each firm have as follows:

$$q_1 = \frac{1}{3}\left[1 - \frac{1 + 2k - 2t(1+k)}{(1-t)}\cdot w\right] \quad (3)$$

$$q_2 = \frac{1 - t - \left(1 - k(1-t) + t\right)w}{3\cdot(1-t)} \quad (4)$$

Therefore, $q_1 - q_2 = \frac{\left(t(1+k) - k\right)}{1-t}\cdot w$, that is, if $t > \frac{k}{(1+k)}$ then $q_1 > q_2$, while if $t < \frac{k}{(1+k)}$ then $q_1 < q_2$. It is clear that if the implied tax rate is high enough (greater than $\frac{k}{1+k}$), then the firm that declares its personnel will enjoy higher market share. Otherwise, if the tax rate is low enough (less than $\frac{k}{1+k}$), then firm 2 enjoys higher market share. So, in terms of market share, we can illustrate the above with the following diagram:



It reveals that each combination of t & k above the curve $t^* = \frac{k}{1+k}$ obliterates any competitive advantage of the second firm, derived from the practice of undeclared labour, since in that case the first firm will enjoy greater market share. On the other hand, if any combination of t and k below the curve is applied, then the second firm will have an incentive to practice undeclared labour, since in this manner it will obtain greater market share.

The same result also applies with profit analysis. The profits of each firm, as they are derived, have as follows:

$$\Pi_1 = \frac{(-1 + t + w + 2\,k\,w - 2\,(1 + k)\,t\,w)^2}{9\,(1 - t)} \tag{5}$$

$$\Pi_2 = \frac{(-1 + t + (1 - k\,(1 - t) + t)\,w)^2}{9\,(1 - t)} \tag{6}$$

Abstracting (5)-(6) we have:

$$\Pi_1 - \Pi_2 = \frac{(k(-1+t)+t)\,w\,(2 - (2+k)\,w + t\,(-2+w+kw))}{3(1-t)} \tag{7}$$

The roots of the above expression are $t_1^* = \frac{k}{1+k}$ and $t_2^* = \frac{-2+2w+kw}{-2+w+kw}$. Since $t_{cr} = \frac{1+(-1+k)w}{1+w+kw} < t_2^* = \frac{-2+2w+kw}{-2+w+kw}$ for 0 < w < 0.5, we reject t_2^* as a critical value[3] and we conclude to the same results, as for the market share analysis; i.e. if $t < t_1^* = \frac{k}{1+k}$ then the firm that practices undeclared labour will gain more profits than the

[3] As already mentioned, ***t*** should be less than t_{cr}.

other one which declares its personnel. If, on the other hand, $t > t_1^*$, then the firm that declares its workers will gain more profits. Preposition 1 summarizes.

Proposition 1:

In the case of exogenous wage, the greater the tax rate than $\frac{k}{1+k}$ *is, the less strong is the incentive for undeclared labour. In other words, comparatively low enough taxation* $\left(less\ than\ \frac{k}{1+k}\right)$ *will create incentives for undeclared labour and conversely.*

2.3. Conclusions

Interpreting the results above, a comparatively low tax rate will enforce the phenomenon of undeclared labour. As a matter of fact, firms face an opportunity cost – dilemma:

- Either they practice undeclared work, pay no contributions for social insurance, but they state more profits and thus pay more taxes
- or they declare their personnel and pay the relevant contributions for social insurance, but they pay fewer taxes due to the fewer profits resulting for taxation.

Any combination of tax / contributions rates under the $t_1^* = \frac{k}{1+k}$ curve will indeed lead firms to practice undeclared labour, in order to avoid paying contributions for social security, since the alternative choice is more costly.

3. Undeclared Labour in Unionized Oligopoly with proportional taxation

3.1. The Model

Consider a homogeneous good market, where two symmetric firms compete by adjusting their quantities. Production exhibits constant returns to scale and requires only labour input to produce the good. Moreover, each firm possesses a Leontief technology, so the capital stock is always sufficient to produce the good.

The production function of each firm can be defined as $q_i = L_i$ (i = 1, 2), where q (L) denotes output (employment), and the productivity of labour is normalized to unity. Moreover, let the inverse demand function specified of the simple normalized linear form, $P(Q) = \alpha - \beta \cdot Q$, where Q is the aggregate output: $Q = q_1 + q_2$.

Firms apply undeclared labour to $a_i \cdot L_i = a_i \cdot q_i$, $0 < a_{i=1,2} < 1$, of their workers. Thus, the cost for undeclared (declared) labour forms as follows: $a_i \cdot w_i \cdot L_i = a_i \cdot w_i \cdot q_i$ $\big((1 - a_i) \cdot w_i \cdot L_i + (k \cdot (1 - a_i) \cdot w_i \cdot L_i) = (1 - a_i) \cdot w_i \cdot q_i + (k \cdot (1 - a_i) \cdot w_i \cdot q_i)\big)$, where *k* stands for the social insurance contribution rate ($0 < k < 1$).

Respectively to undeclared labour, firms apply declared labour $(1 - a_i) \cdot L_i = (1 - a_i) \cdot q_i$, thus the labor cost for declared labour comprises from the wages $(1 - a_i) \cdot w_i \cdot L_i = (1 - a_i) \cdot w_i \cdot q_i$ plus the contributions for social insurance $k \cdot (1 - a_i) \cdot w_i \cdot L_i = k \cdot (1 - a_i) \cdot w_i \cdot q_i$. The total cost for declared labour forms as $(1 + k) \cdot (1 - a_i) \cdot w_i \cdot q_i$.

We also assume proportional direct taxation – rate denoted as *t* – for firms' profit formed as $t \cdot \Big((p \cdot q_i) - \big((1 + k) \cdot (1 - a_i) \cdot w_i \cdot q_i\big)\Big)$. Note here that the taxable profits are calculated by abstracting the cost only for declared labour from i's firm revenues. The cost for undeclared labour remains unknown to the authorities.

Summarizing all the above, the firms' net profit function has as follows:

$$\Pi_i = [p \cdot q_i] - [a_i \cdot w_i \cdot q_i] - [(1 - a_i) \cdot w_i \cdot q_i] - [k \cdot (1 - a_i) \cdot w_i \cdot q_i] - t[(p \cdot q_i) - (1 - a_i) \cdot w_i \cdot q_i - k \cdot (1 - a_i) \cdot w_i \cdot q_i] \quad (8)$$

Firms will choose in the last stage of the game those quantities and that rate of undeclared labour - simultaneously - in order to maximize their profit.

Given risk-neutral fixed membership and immobile labour, according to the utilitarian hypothesis, unions are assumed to maximize rents (for simplicity, we normalize reservation wage to zero, as such a normalization does not qualitatively affect the final state of the equilibrium), reflecting the aggregate labour market preferences of union members. Unions are assumed to be an insider in the labour market, thus having full knowledge of the undeclared labour phenomenon and its size. Assuming, also, proportional taxation for the individuals – employees at the same tax rate *t*, unions' utility comprises from

- the income of the undeclared members $\mathrm{a_i} \cdot w_i \cdot q_i$
- the income of the declared members $(1 - \mathrm{a_i}) \cdot w_i \cdot q_i$
- the cost of social insurance of the declared members, valued as a fringe benefit $k \cdot (1 - \mathrm{a_i}) \cdot w_i \cdot q_i$
- minus the taxation of the declared members $t \cdot (1 - \mathrm{a_i}) \cdot w_i \cdot q_i$.

Summarizing the above, unions' utility function forms as:

$$U_i = [\mathrm{a_i} \cdot w_i \cdot q_i] + [(1 - \mathrm{a_i}) \cdot w_i \cdot q_i] + [k \cdot (1 - \mathrm{a_i}) \cdot w_i \cdot q_i] - [t \cdot (1 - \mathrm{a_i}) \cdot w_i \cdot q_i] \quad (9)$$

Regarding the wage-setting structure, we assume de-facto decentralized wage bargaining regime; each union will negotiate the wage (and thus the employment level) with the relevant firm, considering the maximization of its utility. Unions are moreover assumed to possess a bargaining power of one (monopoly unions) - for simplicity reasons - during labour-management negotiations.

Arising from the above, a three-stage game can be formally addressed as follows:

1. Social Planner sets the optimal tax and social insurance contributions rates.
2. Decentralized wage bargaining takes place, where the wage - and thus the employment – is agreed among firms and unions.
3. Firms determine their quantities in the market (Cournot competition) as well as the optimal level of undeclared labour.

We shall proceed with the further research of the model, using backward induction.

3.1. Solving the model.

Proceeding with the resolution of the model and using backward induction let us consider the third stage of the game first: in the subgame perfect equilibrium (SPE) each firm independently chooses its employment/output level as well as the rate of undeclared labour so as to maximize its profit, given the firm-specific wage contract resulting from Stage 2. Taking first order conditions of the profit functions [8] simultaneously as to quantities and the rates of undeclared labour simultaneously, we derive the optimal output functions.

However, calculating the first order partial differential of the profit function as to a, results:

$$f'(\Pi_i, a_i) = \frac{\Delta(\Pi_i(q_i, a_i)}{\Delta(a_i)} = -(\mathrm{k}\,(-1\ +\ \mathrm{t})\ +\ \mathrm{t})\ q_i \cdot\ w_i$$

We notice that the profit function is monotonic as to a:

$$\begin{cases} f'(\Pi_i, a_i) > 0, \forall\ \ t < \frac{k}{1+k} \\ f'(\Pi_i, a_i) < 0, \forall\ \ t > \frac{k}{1+k} \end{cases}$$

Thus, firms maximize their profit (as to a_i[0,1]), setting

$$\begin{cases} a_i^* = 1\ \ \forall\ \ \ t < \dfrac{k}{1+k} \\ a_i^* = 0\ \ \forall\ \ \ t > \dfrac{k}{1+k} \end{cases}$$

Therefore, the profit functions (derived from function [8]) slit in two forms, depending on the relevant value of *t* as to *k*:

$$[UDL\ case^4]: for\ t < \frac{k}{1+k} \rightarrow\ \Pi_i = \left[p \cdot q_i\right] - \left[w_i \cdot q_i\right] - t\left[(p \cdot q_i)\right] \quad (10)$$

$$\left[DL\ case^5\right]: for\ t > \frac{k}{1+k} \rightarrow\ \Pi_i = \left[p \cdot q_i\right] - \left[(1+k)w_i \cdot q_i\right] - t\left[(p \cdot q_i) - (1+k)w_i \cdot q_i\right] \quad (11)$$

We now take first order conditions of the profit functions [10] & [11] as to

[4] Setting $a_i^* = 0$, means that firms maximize their profit using undeclared labour (UDL) only, ceteris paribus.

[5] Setting $a_i^* = 1$, means that firms maximize their profit using declared labour (DL) only, ceteris paribus.

quantities to derive the optimal output functions.

UDL Case ($t < \frac{k}{1+k}, a_i^* = 1$)	*DL Case* ($t > \frac{k}{1+k}, a_i^* = 0$)	
$q_1 = \frac{\alpha - t\alpha - 2w_1 + w_2}{3\beta - 3t\beta}$	$q_1 = \frac{\alpha - 2(1+k)w_1 + (1+k)w_2}{3\beta}$	(12)
$q_2 = \frac{\alpha - t\alpha + w_1 - 2w_2}{3\beta - 3t\beta}$	$q_2 = \frac{\alpha + (1+k)w_1 - 2(1+k)w_2}{3\beta}$	(13)

Let us therefore proceed to Stage 2 of the game. By virtue of the previous stage and taking first order conditions of unions' utility [9], the following wages are specified:

UDL Case ($t < \frac{k}{1+k}, a_i^* = 1$)	*DL Case* ($t > \frac{k}{1+k}, a_i^* = 0$)	
$w_1 = -\frac{1}{3}(-1+t)\alpha$	$w_1 = \frac{\alpha}{3+3k}$	(14)
$w_2 = -\frac{1}{3}(-1+t)\alpha$	$w_2 = \frac{\alpha}{3+3k}$	(15)

Replacing expressions [14]-[15] into [8]-[9] and solving the game, we have the following final output:

	UDL Case ($t < \frac{k}{1+k}, a_i^* = 1$)	*DL Case* ($t > \frac{k}{1+k}, a_i^* = 0$)	
q_1	$\frac{2\alpha}{9\beta}$	$\frac{2\alpha}{9\beta}$	(16)
q_2	$\frac{2\alpha}{9\beta}$	$\frac{2\alpha}{9\beta}$	(17)
p	$\frac{5\alpha}{9}$	$\frac{5\alpha}{9}$	(18)
Π_1	$\frac{4(1-t)\alpha^2}{81\beta}$	$\frac{4(1-t)\alpha^2}{81\beta}$	(19)
Π_2	$\frac{4(1-t)\alpha^2}{81\beta}$	$\frac{4(1-t)\alpha^2}{81\beta}$	(20)
w_1	$\frac{1-t}{3}\alpha$	$\frac{\alpha}{3+3k}$	(21)
w_2	$\frac{1-t}{3}\alpha$	$\frac{\alpha}{3+3k}$	(22)
U_1	$\frac{2(1-t)\alpha^2}{27\beta}$	$\frac{2(1+k-t)\alpha^2}{27(1+k)\beta}$	(23)

U_2	$\frac{2(1-t)\alpha^2}{27\beta}$	$\frac{2(1+k-t)\alpha^2}{27(1+k)\beta}$	(24)

Continuing our analysis, we further define social revenues and social welfare. Public revenues (R) consist of the contributions for social insurance (R_c) plus the revenues of taxation (R_t), illustrated as below:

$$R_c = \left((1-\mathrm{a}_1)\cdot k\cdot \mathrm{w}_1\cdot \mathrm{q}_1\right) + \left((1-\mathrm{a}_2)\cdot k\cdot \mathrm{w}_2\cdot \mathrm{q}_2\right) \quad (25)$$

$$\mathrm{R}_t = (t\cdot(p\cdot \mathrm{q}_1 - (1-\mathrm{a}_1)\cdot(1+k)\cdot \mathrm{w}_1\cdot \mathrm{q}_1)^2) + (t\cdot(p\cdot \mathrm{q}_2 - (1-\mathrm{a}_2)\cdot(1+k)\cdot \mathrm{w}_2\cdot \mathrm{q}_2)^2) + (t\cdot \mathrm{w}_1\cdot(1-\mathrm{a}_1)\cdot \mathrm{q}_1) + (t\cdot \mathrm{w}_2\cdot(1-\mathrm{a}_2)\cdot \mathrm{q}_2) \quad (26)$$

$$\mathrm{R} = \mathrm{R_c} + \mathrm{R_t} \quad (27)$$

The social welfare (SW) results from the aggregation of the unions' utility, the firms' profits and the consumer surplus (CS). Thus, the derived social welfare appears to be as follows:

$$\mathrm{CS} = \frac{1}{2}(q_1+q_2)^2 \quad (28)$$

$$\mathrm{SW} = \mathrm{U_1} + \mathrm{U_2} + \Pi_1 + \Pi_2 + \mathrm{CS} \quad (29)$$

Substituting the results [16]-[24] to the expressions [25]-[29] and simplifying, we obtain the following results:

	UDL Case ($t < \frac{k}{1+k}, \mathrm{a_i^*} = 1$)	*DL Case* ($t > \frac{k}{1+k}, \mathrm{a_i^*} = 0$)	
R_c	0	$\frac{4k\alpha^2}{27\beta + 27k\beta}$	(30)
R_t	$\frac{20t\alpha^2}{81\beta}$	$\frac{4(5+2k)t\alpha^2}{81(1+k)\beta}$	(31)
R	$\frac{20t\alpha^2}{81\beta}$	$\frac{4(5t+k(3+2t))\alpha^2}{81(1+k)\beta}$	(32)
CS	$\frac{8\alpha^2}{81\beta^2}$	$\frac{8\alpha^2}{81\beta^2}$	(33)
SW	$\frac{4\alpha^2(2+5\beta)}{81\beta^2}$	$\frac{4\alpha^2(2+5\beta+k(2+8\beta))}{81(1+k)\beta^2}$	(34)

3.2. Equilibrium analysis.

Interpreting the structure of the game, let us begin from the last stage of it. Considering the relative value of *t* as to *k* set by the social planner, firms adjust their quantities and the rate of undeclared labour simultaneously to maximize profits. Note here that we treat undeclared labour as it was a legal phenomenon, in order to investigate the economic benefits that its implementation gives. Thus, we do not include any penalties in our analysis.

Since the profit function [8] is monotonic as to a, it proves that if *t* is low enough, lower than $\frac{k}{1+k}$, then firms maximize their profit setting a=1. That means firms will apply UDL for all their workers for relatively low values of *t* as to *k*. Relative results apply in the case that *t* is high enough, higher than $\frac{k}{1+k}$. In this case, firms will maximize their profit declaring all their workers. We should stress out here that as $k \in [0,1] \rightarrow \frac{k}{1+k} \in [0,0.5]$. It results that if *t* is greater than 0.5, then (independently of k) firms maximize their profits using declared labour for all their workers.

Proposition 1 summarizes:

Proposition 1:

If t is lower than a critical value $t^* = \frac{k}{1+k}$, then the implementation of undeclared labour will give more output, employment and firms' profits. If - on the other hand - t is greater than the critical value t^* mentioned above, the opposite state applies. Last, if *t* is great enough, greater than 0.5, independently of k, then declaring labour will maximize output, employment and profits.

Interpreting the above proposition, there is an opportunity cost between taxes and social security contributions. If a relatively great tax rate *t* applies in the economy, greater than a critical value $t^* = \frac{k}{1+k}$ or greater than 0.5, then firms will pay social security contributions (declaring their workers), in order to declare more expenses and fewer profits, reducing by this way the taxes payable. On the other hand, adjusting the tax rate t less than 0.5 & t*, will motivate firms to apply UDL, in

order to maintain their profits. Note also that under $t < t^* = \frac{k}{1+k}, 0.5$ UDL will maintain the same level of employment.

Proceeding to the second stage, unions will adjust their wages (and thus employment), depending on the firms' decision about the level of a. Note that, under the risk of non-employment of their members, unions will not denounce any UDL phenomenon and thus they will consent silently by just adjusting the wages on each case. This mechanism proofs both subgames consistent and thus Nash subgames perfect.

Abstracting [21UDL] minus [21DL]:

$$w_{a=1} - w_{a=0} = \frac{(k(1-t)-t)\alpha}{3(1+k)} \rightarrow \begin{cases} w_{a=1} - w_{a=0} > 0 \rightarrow w_{a=1} > w_{a=0}\,, for\ t < \frac{k}{1+k} \\ w_{a=1} - w_{a=0} = 0 \rightarrow w_{a=1} = w_{a=0}\,, for\ t = \frac{k}{1+k} \\ w_{a=1} - w_{a=0} < 0 \rightarrow w_{a=1} < w_{a=0}\,, for\ t > \frac{k}{1+k} \end{cases}$$

In addition, abstracting [23UDL] minus [23DL]:

$$U_{a=1} - U_{a=0} = -\frac{2kt\alpha^2}{27\beta + 27k\beta} < 0$$

Proposition 2 summarizes:

Proposition 2:

Through the bargaining process, labour unions maximize the wages in each case (either $t > t^*$, or $t < t^*$). However, unions' utility is greater in the Declared Labour Case rather than the Undeclared Labour Case.

In each case, either a=0 or a=1, the wages prove to be optimal, maintaining employment unchanged. Nevertheless, it proves that unions value the social security benefit more than the taxation imposed on their (declared) wages. Thus, unions' utility is greater when their members are properly declared and insured.

Regarding the consumers, abstracting [33UDL] minus [33DL]:

$$CS_0 - CS_1 = \frac{8\alpha^2}{81\beta^2} - \frac{8\alpha^2}{81\beta^2} = 0$$

Proposition 3 summarizes:

Proposition 3:

Consumers remain indifferent to the Undeclared Labour Phenomenon as Consumer Surplus is calculated unchanged in each instance.

Firms use Undeclared Labour to maintain the same amount of profit. As the employment remain unchanged (through the process of wage bargaining), firms' output q also remains stable. Furthermore, we notice that price [18] is the same in both instances. Therefore, Consumer Surplus remain also unchanged. Thus, under strictly economic criteria, we conclude that consumers remain indifferent to the phenomenon of undeclared work.

Finally, in terms of Social Welfare, we abstract [34UDL] minus [34DL]:

$$SW_0 - SW_1 = \frac{4\alpha^2(2+5\beta)}{81\beta^2} - \frac{4\alpha^2(2+5\beta+k(2+8\beta))}{81(1+k)\beta^2} = -\frac{4k\alpha^2}{27\beta+27k\beta} < 0 \rightarrow SW_0 < SW_1$$ [6]

Proposition 4 summarizes:

Proposition 4:

Society enjoys greater prosperity applying declared labour in the labour market rather than applying undeclared labour.

As profits and consumer surplus remain unchanged in both instances, the increase of social welfare is solely due to the increased unions' utility.

As declared labour increases social welfare, a benevolent social planner should adjust his policy properly. From the model analysis results that if the social planner set a tax rate $t > t^* = \frac{k}{1+k}$, then the labour market will be regulated to declared labour, as the financial incentives to apply undeclared labour by this way will have been eliminated. In this case, setting the appropriate tax rate in relation to the respective social security contribution rate is proved to be an effective policy tool in order to confront undeclared labour.

[6] We define SW as the sum of firms' profit, unions' utility and consumer surplus, under the closed budget assumption: taxes are returned to the community either by public investments or by the provision of public goods. Nevertheless, the qualitive results remain the same as well as we embed the revenues of the social planner coming from taxation in the SW function.

In the case that due to fiscal adjustment purposes the social planner is granted with limited ability setting the proper tax rate, an alternative policy should apply. An effective policy tool could include the subsidy of the unions, in order to regain the loss of their utility. In this way, social planner will maintain social welfare at the same levels.

Proposition 5 summarizes:

Proposition 5:

In a proportionate tax system, where firms and workers face the same tax rate t, setting a proper tax rate $t > t^* = \frac{k}{1+k}$ forms an effective policy tool to confront undeclared labour.

In the case that setting the above proper tax rate is prohibited, an alternative effective policy tool may include the subsidy of the unions to regain the loss of their utility.

4. Conclusion

Undeclared labour constitutes a complex phenomenon, where tax evasion and social security fraud are involved. Both employers and employees voluntarily collude, because of the potential gain in avoiding taxes and social security contributions, social rights and the cost of complying with regulations.

As it concerns our present research, we introduced a theoretical 3-staged model that endogenizes undeclared labour and analyzes the phenomenon within I/O framework. We endogenized the selection of the optimal rate of undeclared labour from the firms - simultaneously with the quantities. Furthermore, model's assumptions include proportional taxation for both firms and workers. To reveal the side effects that undeclared labour creates to the society, we examined both firms and unions objectives considering UDL as a legal selection. Furthermore, we focused on the reducing of the financial attractiveness of undeclared labour. For this reason, we have not included any forms of surveillance and sanction mechanisms (penalties, fines etc) in our analysis.

The findings of our analysis evince that firms may apply undeclared labour as a tool to maintain their profits unchanged, under relatively low taxation ($t < t^* = \frac{k}{1+k}$). Furthermore, we proved that while consumers remain indifferent to the UDL phenomenon, unions enjoy greater utility under declared labour state. Finally, we propose that - since social welfare is greater in DL state rather than UDL state - setting a proper tax rate $t < t^* = \frac{k}{1+k}$ consists an effective policy tool to confront undeclared labour. In the case that setting the above proper tax rate is prohibited, an alternative effective policy tool may include the subsidy of the unions to regain the loss of their utility.

Since the project has not any relative research background, possible extensions of this research may be yet quite more promising. Further research may include different taxation systems, different types of competition (e.g. Bertrand Competition), different types of wage bargaining (e.g. centralized bargaining, non-monopoly unions), endogenization of state's interference in labour market (e.g. screening for undeclared labour) and a cost-benefit analysis for the determination of

the optimal governmental surveillance's cost or the social's optimal rate of undeclared labour. The forthcoming research will comprise a key role for us to acquire a spherical knowledge of the undeclared labour phenomenon and its side effects.

5. Appendix

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