

# Three-Dimensional Field-Flux Eigenmode Formulation for Periodic Graphene Structures

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## Introduction

- ✓ A precise 3-d Field-Flux FEM Formulation for Periodic GRAPHENE structures is implemented
- ✓ The formulation is spurious free and results in a linear Eigenvalue problem. Allows the examination of the wave inside passbands and stopbands of periodic Graphene structures.
- ✓ All computed Fields are of the same order of approximation

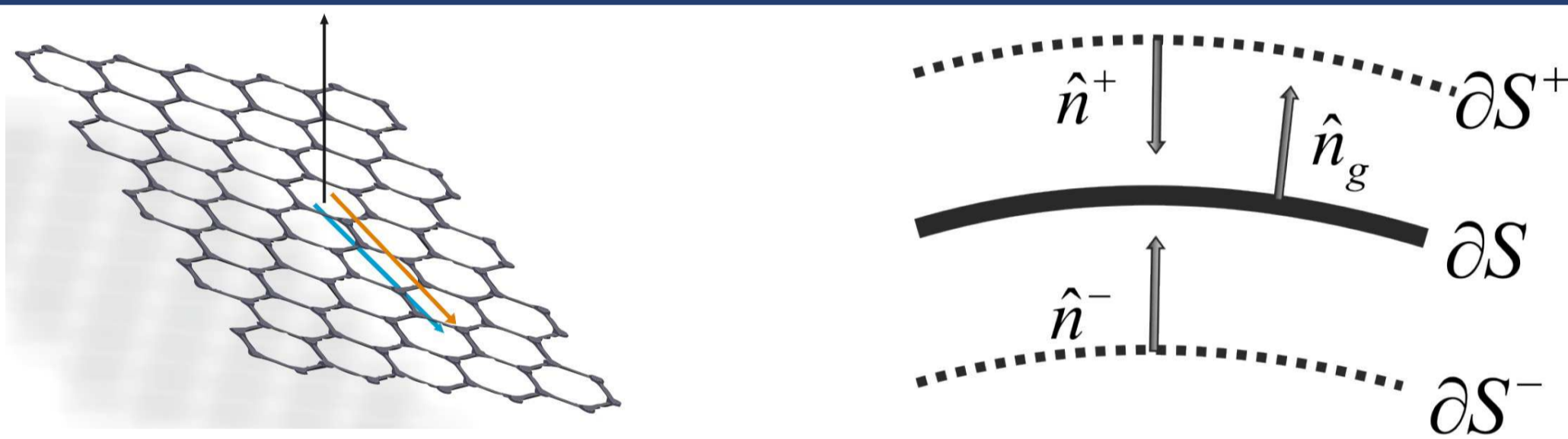
## Field-Flux Eigenmode Formulation

$$\begin{aligned} \nabla \times \mathbf{E} &= -j\omega \mathbf{B} & \mathbf{\tilde{B}} &= -jc_0 \mathbf{B} & \nabla \times \mathbf{E} &= k_0 \mathbf{\tilde{B}}, \\ \nabla \times \bar{\mu}^{-1} \mathbf{B} &= j\omega \epsilon_0 \bar{\epsilon}_r \mathbf{E} & \text{Scaling of } \mathbf{B} & \text{to avoid ill-conditioning of the final system of equations} & \nabla \times \bar{\mu}_r^{-1} \mathbf{\tilde{B}} &= k_0 \bar{\epsilon}_r \mathbf{E} \\ \mathbf{E} &= \mathbf{e} e^{-jk_0 n_{eff} \hat{\mathbf{k}} \cdot \mathbf{r}}, & \text{Transformation of Maxwell's equations according to Bloch-Floquet theorem} & & \nabla \times \mathbf{e} - jk_0 n_{eff} \hat{\mathbf{k}} \times \mathbf{e} &= k_0 \mathbf{\tilde{b}}, \\ \mathbf{\tilde{B}} &= \mathbf{\tilde{b}} e^{-jk_0 n_{eff} \hat{\mathbf{k}} \cdot \mathbf{r}} & & & \nabla \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}} - jk_0 n_{eff} \hat{\mathbf{k}} \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}} &= k_0 \bar{\epsilon}_r \mathbf{e} \end{aligned}$$

$$\begin{aligned} \iiint_{\Omega} \mathbf{\tilde{b}}' \cdot \nabla \times \mathbf{e} dv - k_0 \iiint_{\Omega} \mathbf{\tilde{b}}' \cdot \mathbf{\tilde{b}} dv - jn_{eff} k_0 \iiint_{\Omega} \mathbf{\tilde{b}}' \cdot \hat{\mathbf{k}} \times \mathbf{e} dv &= 0, \\ \iiint_{\Omega} \nabla \times \mathbf{e}' \cdot \bar{\mu}_r^{-1} \mathbf{\tilde{b}} dv + \iint_{d\Omega} \mathbf{e}' \cdot \hat{\mathbf{n}}_{ext} \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}} ds - k_0 \iiint_{\Omega} \mathbf{e}' \cdot \bar{\epsilon}_r \mathbf{e} dv &= 0, \\ -jn_{eff} k_0 \iiint_{\Omega} \mathbf{e}' \cdot \hat{\mathbf{k}} \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}} dv &= 0 \end{aligned}$$

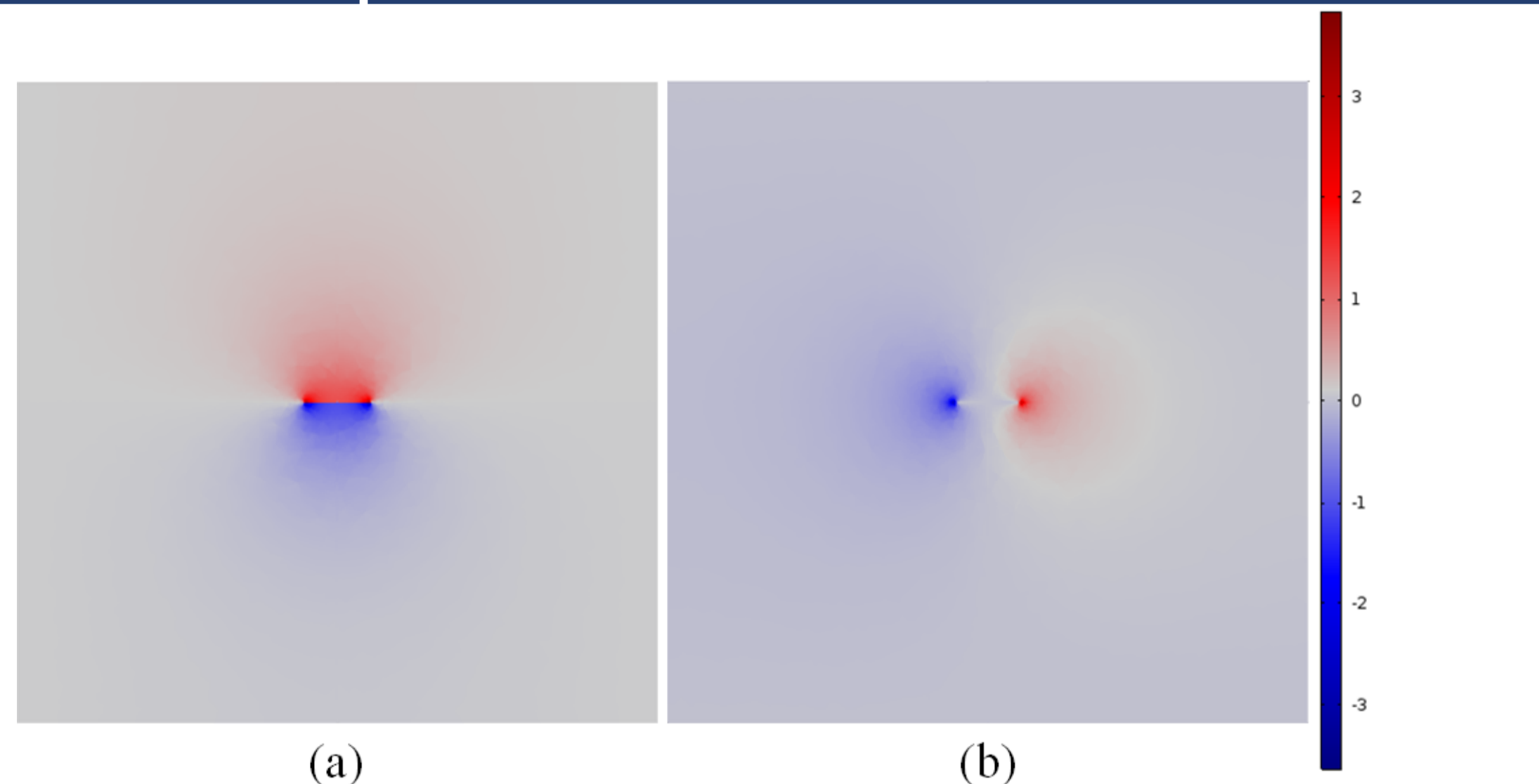
Final FEM Galerkin Formulation

## Implementation of Graphene

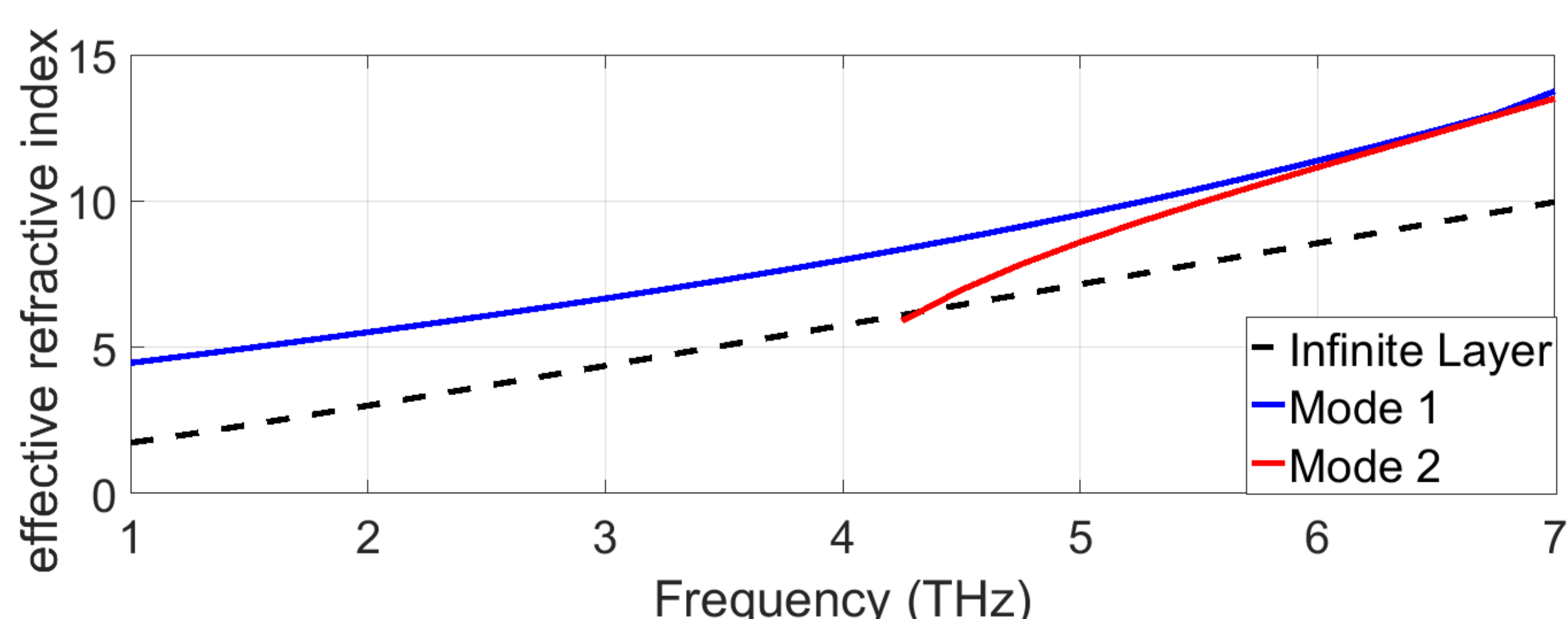


- ✓ Graphene is treated as a finite conductivity boundary
- $$\hat{\mathbf{n}}_g \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}}^+ - \hat{\mathbf{n}}_g \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}}^- = -j\eta_0 \sigma_g \mathbf{e}_{tg}$$
- ✓ The Graphene's Surface is considered as an exterior boundary
- $$\iint_{S_g} \mathbf{e}' \cdot (\hat{\mathbf{n}}_{ext} \times \bar{\mu}_r^{-1} \mathbf{\tilde{b}}) ds = j\eta_0 \iint_{S_g} \mathbf{e}'_{tg} \cdot \sigma_g \mathbf{e}_{tg} ds$$

## Computational Results I : 5μm Free-Standing Graphene Microribbon

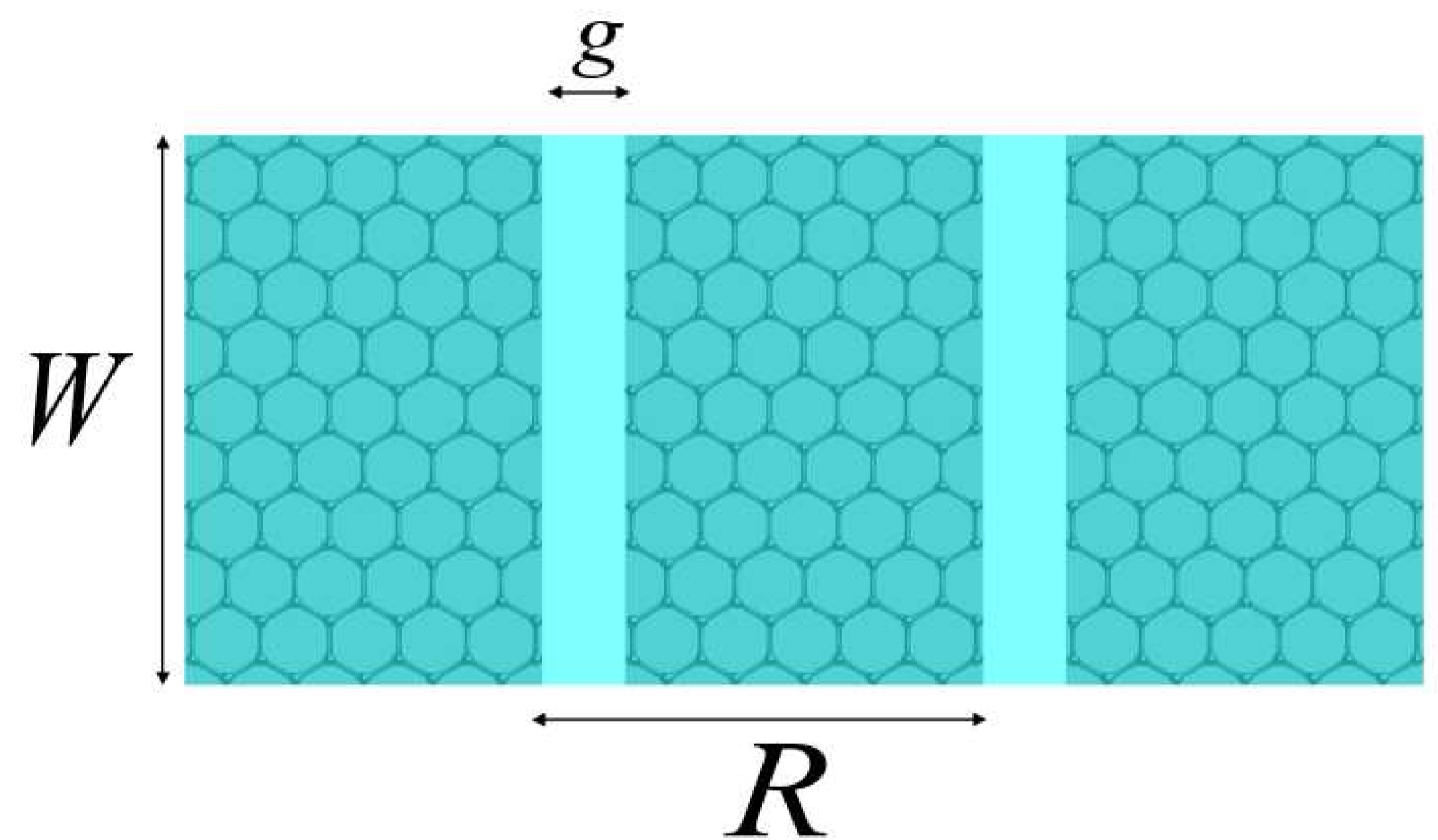


Distribution of the normal (a) and tangential (b) electric field at the transverse plane on the propagation axis of the first mode at 1 THz.



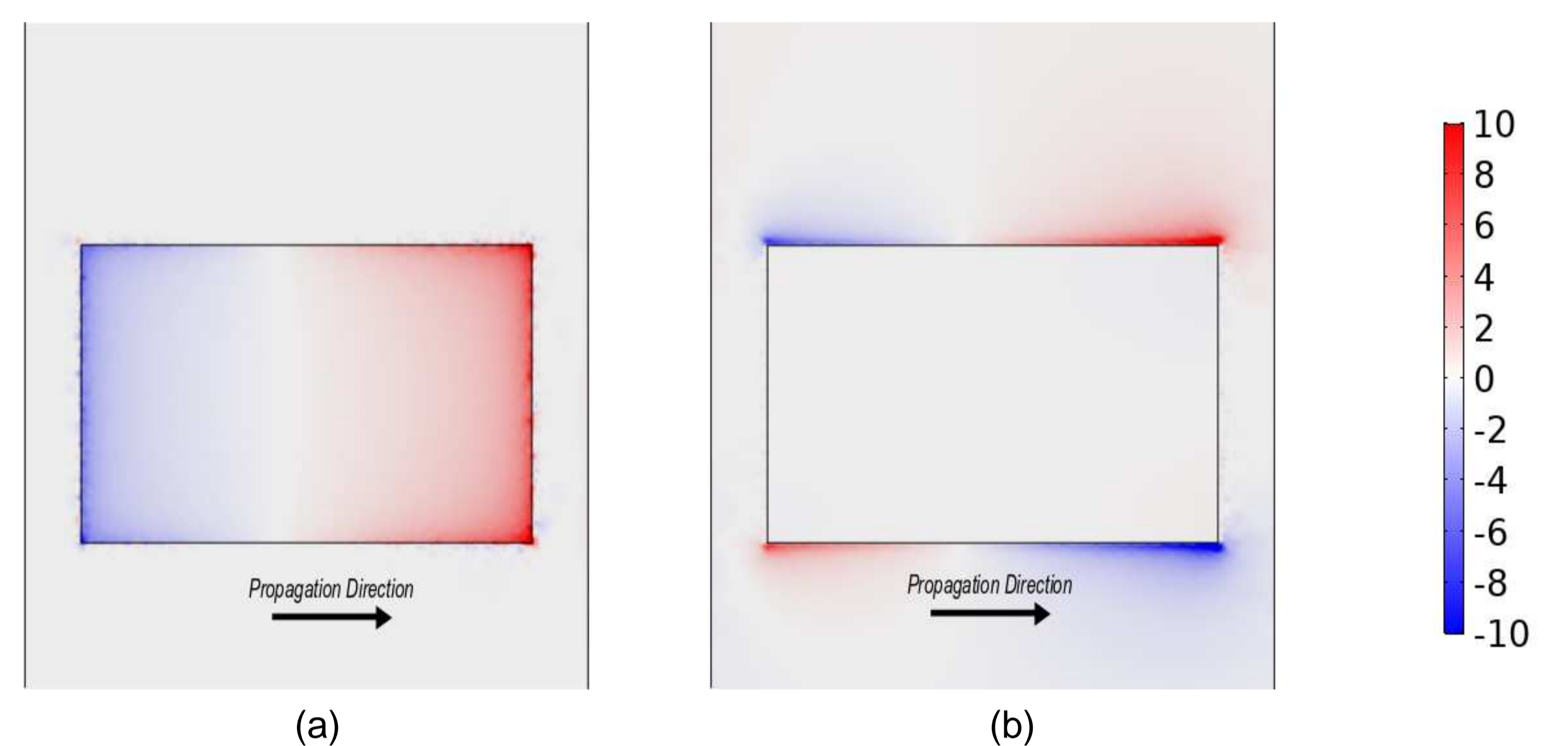
Dispersion curves of the first two modes of a 5μm graphene microribbon compared to the infinite layer [2].

## Computational Results II : Periodic Graphene Structure



Periodic Arrangement of Graphene Microstrips

$$W = 5\mu\text{m} \quad g = 2\mu\text{m} \quad R = 10\mu\text{m}$$



Distribution of the normal (a) and tangential (b) electric field on the plane of the Graphene structure at 2 THz.

## References

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