Single- and Multi-Channel Nonlinear Phenomena in Resonant Structures Comprising Graphene

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Abstract **– Graphene exhibits high third-order nonlinearity in both THz and NIR regimes. Here, we consider single- and multi-channel nonlinear phenomena in resonators comprising graphene to exploit its high nonlinearity. Specifically, we study optical bistability in the NIR with a graphene-covered silicon slot microring resonator and degenerate four-wave mixing in the THz regime with a graphene ribbon resonator. For calculating the nonlinear response, we employ a recently developed perturbation theory/coupled mode theory framework which is both efficient and accurate. We find low power requirements and high conversion efficiencies, highlighting graphene's potential for nonlinear applications.**

I. Introduction

In recent years, 2D photonic materials have drawn considerable interest for an abundance of linear and nonlinear applications. Graphene, the most well-studied and acclaimed 2D material, gathers several attractive features: it supports strongly localized surface plasmon polaritons in the THz, it can offer wide tunability by means of electric biasing, and it is becoming technologically mature offering the ability of being transferred/deposited on different substrates and being compatible with the silicon-on-insulator (SOI) photonic platform. Importantly, graphene exhibits high third-order nonlinearity in both NIR and THz regimes: recent measurements [1] indicate values of the equivalent nonlinear refractive index n_2 (when graphene is perceived as an equivalent bulk medium) at least in the order of 10^{-13} m^2/W in the NIR and even stronger in the THz, surpassing conventional nonlinear materials (semiconductors, nonlinear polymers, chalcogenide glasses) by orders of magnitude.

Here, we present our recent work on graphene-enhanced nonlinear resonant systems [2-5], covering both the THz and NIR regimes. More specifically, we construct a systematic mathematical framework based on perturbation theory and temporal coupled mode theory (CMT) that can describe single-channel (e.g. self-phase modulation, bistability, self pulsation, saturable absorption) and multi-channel nonlinear phenomena (e.g., cross-phase modulation, four-wave mixing, third harmonic generation) in resonant structures comprising graphene. Our theoretical framework extends existing literature on bulk nonlinear materials [6-9] by incorporating dispersive conductive *sheet* materials such as graphene, naturally modeled as infinitesimally thin and described with a complex *surface* conductivity [measured in S and $S(m/V)^2$ for linear and third-order nonlinear terms, respectively]. This is achieved by including a nonlinear surface current term next to the more common nonlinear bulk polarization term. In deriving the framework, we find that media with a complex conductivity disturb the equality of electric and magnetic energies on resonance [2], a condition which is typically taken for granted. This stems from the reactive power associated with the imaginary part of graphene's surface conductivity. In addition, we find that the dispersive nature of conductive materials must be always considered, otherwise the energy stored in the current distribution is erroneously zeroed out.

In Sect. II, we study optical bistability (BI), a single-channel nonlinear phenomenon, in the NIR with a silicon slot microring resonator overlaid with a uniform graphene sheet. Next, in Sect. III we focus on the multi-channel phenomenon of four-wave mixing (FWM) with a graphene ribbon standing wave resonator operating in the THz regime. In the latter case, graphene offers its nonlinearity and additionally provides the waveguiding mechanism in the form of surface plasmon polaritons. In both cases, the graphene-enhanced nonlinear component exhibits excellent performance, a power threshold < 100 mW for BI and a conversion efficiency that can reach -11.3 dB (7.4%) for FWM, indicating the potential of graphene for nonlinear applications.

II. Bistability With Graphene-Covered Silicon Slot Microring in the NIR

In this section, we apply the developed framework to the single-channel phenomenon of optical bistability. The accuracy of the framework in this nonlinear effect has been validated by comparing with full-wave nonlinear finite element method simulations revealing excellent agreement [2]. The structure under study is depicted in Fig. 1(a) and consists of a graphene-covered silicon slot microring resonator (R = 3.25 μm) operating in the NIR (1.55 μm), side coupled to an access waveguide. The silicon-slot platform is judiciously chosen to provide tight field confinement in the slot and maximum interaction with graphene via the tangential electric field components. By optimizing the geometric parameters of the structure, we can obtain a hysteresis loop at low input powers, less than 100 mW [Fig. 1(b)], with a high extinction ratio (ER) of 45 dB. Depending on the history of the system, we can access either of the bistable states A and A', thus offering the possibility for all-optical switching applications. In Fig. 1(c) we demonstrate memory operation by Set and Reset pulses, toggling the system between states A and A'. Importantly, the system settles at the new state in less than 30 ps, mostly due to the free-carrier recombination rate in silicon, allowing for switching rates in the order of 20 GHz. The path followed on the hysteresis loop is shown color-coded in Fig. 1(b).

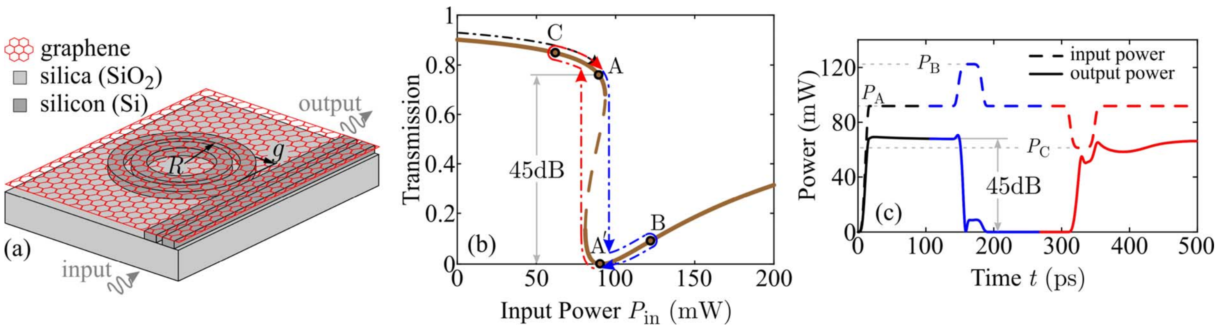

Fig. 1. (a) Graphene-covered silicon slot microring resonator (R = 3.25 μm) operating in the NIR (1.55 μm), side coupled to an access waveguide. (b) Nonlinear CW response, revealing hysteresis and the formation of a bistability loop. For a low input power less than 100 mW, we access bistable states A/A' with a high extinction ratio (ER) of 45 dB between them. (c) Memory switching dynamics revealed by Set and Reset pulses, toggling the system between states A and A'. The route on the bistability curve is marked color-coded in panel (b).

III. Four-Wave Mixing with Graphene Ribbon in THz

Next, we apply the developed framework to the multi-channel phenomenon of degenerate four-wave mixing, involving three wavelengths: pump (ω_1), signal (ω_2) and idler (ω_3). The accuracy of the framework in this nonlinear effect has been validated in [5] by comparing against full-wave nonlinear finite element method simulations. Here, we study a graphene-plasmon standing wave (SW) resonator operating in the THz regime, Fig. 2(a). Considering a w = 1 μm graphene ribbon, we find the optimum resonator length L = 24.3 μm, supporting the 7th, 8th, and 9th order modes at f_3 = 4.58 THz, f_1 = 5 THz, and f_2 = 5.36 THz, respectively. The graphene ribbon resonator can be directly coupled to either one or two feeding waveguides. The conversion efficiency (CE) for the two cases is depicted in Fig. 2(b) and Fig. 2(c), respectively, as a function of the pump (P_p) and signal (P_s) input power. The single waveguide system is found superior, exhibiting an optimum CE, defined as $P_{idler}/(P_p+P_s)$, of -25.7 dB for remarkably low input powers P_p = 172 μW and P_s = 71 μW. It should be noted that the Kerr-induced self and cross frequency shifts impair the performance by detuning the resonances from the operating wavelengths (when set at the unperturbed resonance frequencies), thus lowering the CE. By pre-shifting the operating frequencies to better match the perturbed resonance frequencies, the CE can reach extremely high values up to -11.3 dB (7.4%).

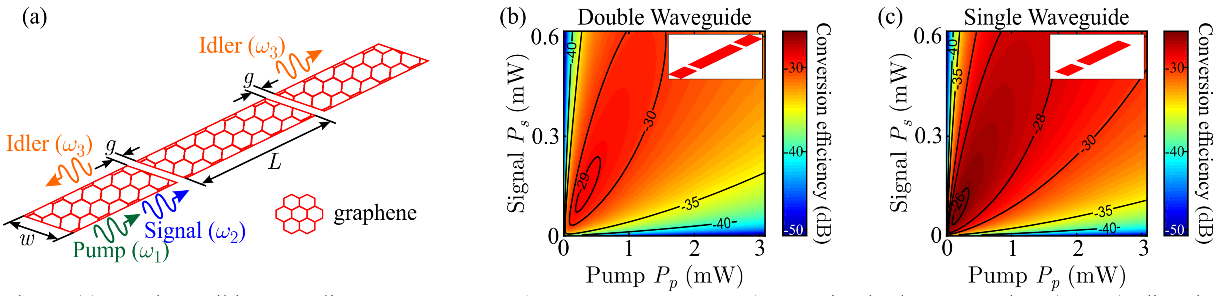

Fig. 2. (a) Graphene ribbon standing wave resonator (w = 1 μm, L = 24.3 μm) operating in the THz regime (5 THz), directly accessed with feeding waveguide(s). (b) Conversion efficiency of the FWM process for the system with two access waveguides shown in panel (a). (c) Conversion efficiency of the FWM process for the system with a single access waveguide (see inset). The latter is found superior.

IV. CONCLUSION

In conclusion, we have applied a recently-developed theoretical framework for analyzing single- and multi-channel nonlinear processes in resonant structures comprising graphene [2-5] to two cases of practical importance: (i) optical bistability in the NIR (Sect. II) and (ii) degenerate four-wave mixing in the THz regime (Sect. III). The different characteristics and the performance evaluation of the two systems are summarized in Table I. As seen by the two cases, graphene-enhanced resonators exhibit low power requirements and high conversion efficiencies, thus exhibiting high potential for practical nonlinear applications.

Table I. Summary of the two systems studied in Sections II and III.

Section	**Nonlinear Effect**	**Number of channels**	**Frequency Regime**	**Material System**	**Resonator Type**	**Coupling Scheme**	**Performance**
II	Bistability	single-λ	NIR	Graphene on SOI	Travelling wave	Side coupling	Input Power: < 100 mW
III	Four wave mixing	multi-λ (three)	THz	Graphene	Standing wave	Direct coupling	Conversion Efficiency: 7.4%

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