Applications of computational techniques on aspects of Social Choice Theory

Post-doctoral report

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Abstract

During this post-doctoral research we focus on applying computational techniques for solving problems and issues of Social Choice Theory. More precisely, we deepen on problems arising in collective decision-making under a social choice theoretic approach. That is, we adopt methods and techniques that lie on aspects of social choice theory for aggregating preferences with aim to aid collective decision making. In addition, we investigate the way of enriching the classical social choice approach by examining the effect of reasoning in the preferences of the group when a collective decision is to be made.

One main aspect of social choice theory is preference aggregation, which in general deals with the aggregation of individual agents' preferences into a collective preference. In a more specific context, a prominent topic of social choice is voting theory, which constitutes a form of preference aggregation. The central problem of voting theory is the computation of the winning alternative in an election when we have as input the preferences of the voters, i.e., the agents. In the literature many voting rules have been proposed in order to aggregate the agents' preferences having as an objective the computation of the winner of the elections. Voting theory is a seminal subject in the computational social choice theory with applications in the society as we can see that voting rules are widely used in collective decision making. For example, voting rules are used in government and municipal elections, groups and committees for taking decisions, voting polls across the internet, etc. During our research we focus on aspects of collective decision-making when social choice methods are applied on a group of decision makers, which we call them as agents.

We start by considering collective decision making and preference aggregation mechanisms, which take as input the preferences of the agents as well as the provided reasoning for these preferences. Our research hypothesis is that a decision made by a group of agents understanding the qualitative rationale behind each other's preferences has better chances to be accepted and used in practice. We follow an algorithmic approach and propose two collective decision making methods which combine argumentation and computational social choice techniques. The first one is a novel qualitative decision process while the other method is a quantitative one. For the qualitative approach we prove theoretical results which show that it can overcome some of the social choice deficiencies. In the quantitative method we quantify the deliberation phase by defining a new voting argumentation framework and its acceptability semantics. We prove theoretical results for these semantics regarding well-known properties that appear in argumentation and social choice theory.

Next, we focus on the problem of multi-criteria decision making, where the goal is to reach an acceptable collective decision aggregating agents' preferences expressed over multiple criteria. We provide a novel modelling of the multi-criteria decision making problem as an inconsistent knowledge base, and we explain how to benefit from the reasoning capabilities of existential rules. The repairs of this knowledge base represent the maximally consistent point of views and inference strategies can be used for decision making.

In the last phase of our research we turn our attention in real decision making applications by implementing aggregation algorithms following a visualizing approach. First, we design a decision-making software tool that applies in decision problems of agricultural engineering. The tool uses methods of computational social choice and argumentation for preference aggregation and collective decision making. Hence, the tool's architecture is composed of two main systems, i.e., the social choice system and the deliberation system. We mainly focus on the social choice system where its implementation is oriented towards practicality, so that it can be applied to different decision problems used in agriculture and are related with the valorization of materials. We complete the picture by providing the implementation of all the algorithms used and needed to support the theoretical results of this research. These algorithms consist the core of future work regarding software tools that use reasoning based methods for collective decision making.

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Chapter 1

Introduction

In the recent years, there has been an increasing interest in the computer science community for problems arising from Social Choice Theory and Decision-making. Social choice methods have been extensively used for taking a decision in a collective group setting. The study towards social choice for collective decision-making had been triggered from the requirement for social justice and fairness in decisions, but also due to the extensive use of voting and elections in various areas of human activity where a decision is to be made. Nowadays, we see that complex voting rules are used in various countries around the world for parliament or municipal elections in order to decide the winning candidates, which will be referred as alternatives[1]. The complex structure of social choice methods (including voting rules) has led to the study of computational techniques and their application on decision problems. On the other hand, argumentative approaches have been also studied for helping decision making. In the argumentative approach the objective is the selection of the best decision with respect to reasoning and rationality. It is our belief that combining these two fields can lead to novel and significant decision making techniques. Therefore, in this research we deepen on decision-making problems under a Computational Social Choice (COMSOC) and Argumentative perspective. The structure of the post-doctoral research report is as follows.

- In the first (this) chapter, we analyze the correlation between the scientific fields of social choice theory and computer science, through which a new field, called computational social choice, emerges. The aim of this chapter is to explain the major problems that are being investigated in this new field, and what is its connection to collective decision making. In this context, we analyze some important voting rules presented in the bibliography and state how a collective decision can be derived from them. In addition, we present a brief introduction on argumentation, which can be also used as an approach that aids decision making.

[1] In AI and Decision-making literature, it is more common to use the term alternative to denote a candidate

- In the second chapter we study decision problems where the outcome has to be based on the agents' preferences as well as the reasoning behind the preferences. We explore how the agents' rationale can be formulated inside the classical voting setting. Therefore, we propose two decision-making algorithmic procedures based on argumentation and social choice (for preference aggregation) which permit us to explore the effect of reasoning and deliberation along with voting for the decision process. The first method is rather qualitative, and show that it can overcome some of the social choice deficiencies. In the second one, we quantify the reasoning phase by defining a new voting argumentation framework and its acceptability semantics. For this method, we prove theoretical results regarding well-known properties coming from argumentation and social choice theory.

- In the third chapter we present the collective decision-making problem from a logic and knowledge base perspective. We model the problem as an inconsistent knowledge base and we infer that the repairs of this knowledge base can be used to aid decision making since they represent the maximal consistent sets of point of views.

- The fourth chapter deals with software implementation of preference aggregation methods, which are depicted by algorithms, that are popular in social choice theory. First, a decision-making software tool for agricultural applications is presented. It uses methods from computational social choice for the computation of the results and Information Visualization techniques for the interface. The objective of the presented software is its application in real use-cases and decision problems. Second, we present all the algorithms used for decision making which support the theoretical research based on social choice and argumentation.

1.1 An intro to Computational Social Choice

Social Choice Theory is a theoretical framework that studies the analysis of methods for making collective decisions or reaching a social welfare by taking into account individual opinions, preferences or interests. In recent years, computer science and artificial intelligence (AI) have been playing an increasingly active role in social choice, and two different research directions have been developed in order to obtain a new subfield called *Computational Social Choice*. The first one concerns the introduction of concepts and methods from AI to solve problems that originate from social choice theory. A starting point for this was the fact that social choice theorists have focused on introducing abstract results related to the existence of procedures that answer specific queries, but the computational issues associated with them have not been sufficiently analyzed. For example, while it is not possible to design a voting protocol that makes it impossible for an agent to cheat in one way or another, it can be shown that cheating successfully is a computationally hard problem, which is considered desirable in practice. This shows AI's contribution and, in general, the impact of computer science on solving problems from social choice theory. In addition to the theoretical analysis of the complexity of the voting protocols, other prominent examples in computational social choice are the formal definition and verification of social procedures (such as fair allocation) using mathematical logic, as well as the adaptation of processes developed from logic and AI in the conventional representation of preferences in combinatorial domains (such as the debate on indivisible goods or voting for committees).

The second line of research in Comsoc has an opposite direction as it deals with introducing social choice theoretic concepts and procedures to solve problems arising in Computer Science and AI. Application of social choice methods are widely used in the development of ranking systems used by search engines, in recommender systems, as well as in decision-making for multi-agent systems.

Nowadays, there has been noted a general tendency for interdisciplinary research that includes, on one hand, decision theory, game theory, social choice and welfare economics, and on the other hand computer science, artificial intelligence, and computational logic. A positive aspect is that the beneficial impact of research on game theory and computer science is already recognized and leads to significant progress in areas such as combined auctions, mechanism design and applications in e-commerce. There are two distinct guidelines according to which we could classify the topics examined in computational social choice:

1. The nature of the social choice problem considered and
2. The type of the computational technique being studied.

These two dimensions are independent to some extent. We give a list of the most important topics following the first guideline:

- **Preference aggregation:** The aggregation of preferences means mapping a collection $P =< P_1, ..., P_n >$ of preference relations of individual agents into a collective preference relation P^*. Sometimes we are only concerned with determining a socially preferred alternative, or a subset of socially preferred alternatives rather than a full collective preference relation: a social choice function maps a collective profile P into a single alternative, while a social choice correspondence maps a collective profile P into an nonempty subset of alternatives. This first topic is less specific than the following ones, which mostly also deal with some sort of preference aggregation, but each in a much more specific context.
- **Voting theory:** Voting is one of the most popular ways of reaching collective decisions. Researchers in social choice theory have studied extensively the properties of various families of voting rules, but have typically neglected computational issues. A whole panorama

of voting rules has been proposed in the literature and we can only mention here a few examples. A scoring rule computes a score (a number) for each alternative from each individual preference profile and selects the alternative with the maximum sum of scores. The plurality rule, for instance, gives score 1 to the most preferred alternative of each agent and 0 to all others. The Borda rule assigns scores from m (the number of alternatives) down to 1 to the alternatives according to their position in the agent's preference. Another important concept is that of the winner according to Condorcet method, i.e., an alternative preferred to any other alternative by a strict majority of agents.

- **Resource allocation and fair division:** Resource allocation of indivisible goods aims at assigning items from a finite set R to the members of a set of agents N, given their preferences over all possible bundles of goods. We have two types of allocation problems, i.e., centralized and distributed. In the former, the assignment is determined by a central authority to which the agents have given their preferences beforehand. In the latter agents negotiate, communicate their interests, and exchange or trade goods in several rounds, possibly in a multilateral manner. We have two types of criteria when assessing the quality of a resource allocation, namely efficiency and fairness. The most fundamental efficiency criterion is the one by Pareto: an allocation should be such that there is not alternative allocation that would be better for some agents without being worse for any of the others. An example for a fairness condition is envy-freeness: an allocation is envy-free if and only if no agent would rather obtain the bundle held by one of the others.

- **Coalition formation:** There are many occasions where agents do not compete but instead cooperate, e.g., to fullfill more efficiently a given task. Suppose for instance that agent x is rewarded 10 when she performs a given task alone, while agent y gets 15. Now if they form a team, the gain is up to 40 (think for instance of two musicians, playing either solo or in a duet). Coalition formation studies typically two questions: what and how coalitions will form for a given problem, and how should then the surplus be divided among the members of the coalition (after they have solved their optimisation problem). Central here is the notion of stability: an agent should have no incentive to leave the coalition. These questions are studied in the field of cooperative game theory, and different solution concepts have been introduced. For instance, the strongest of these, known as the core, requires that no other coalition could make its members better off.

- **Judgement aggregation:** This field aims at studying how a group of individuals should aggregate their members' individual judgements on some interconnected propositions into corresponding collective judgements on these propositions. Such aggregation problems occur in many different collective decision-making bodies (especially committees and expert panels).

In the second line of our proposed taxonomy of topics, i.e., the classification according to the technical issues, we can have the following list of issues:

- computationally hard aggregation rules
- social choice in combinatorial domains
- computational aspects of strategy-proofness and manipulation
- distributed resource allocation and negotiation
- communication requirements in social choice
- logic-based analysis of social procedures
- analysis of social desirable properties on aggregation methods

1.1.1 Correlation between Social Choice and Decision Making

Most of the social choice topics discussed above are connected to collective decision making since in a broader sense the goal of aggregating individual preferences to a collective one can be seen as the decision outcome taking into account the agents' parameters. We will focus on the connection between voting theory and (multi-criteria) decision making. We should start by clarifying the main aspects of voting theory that first appeared at the end of the 18th century under the influence of the French Revolution. It was that period where many systems were proposed that aimed at the fair proclamation of the winner of a voting procedure (i.e., an election). Most of the methods focus on properties of voting systems for general elections or decision-making in committees [26].

The first voting systems were proposed by Borda [43] and Marquis de Condorcet [40] who formulated their respective methods. For a long time there has been no relevant research on social choice systems. The field started to gain attraction again in the 50s with the prominent work of K. Arrow [13]. In the late 20th century, the emergence of large-scale applications for information retrieval and classification set the voting systems in the computer science's research agenda. This is because problems such as the problem of ranking of sets can be considered as a voting problem. In group ranking problems, we are given a set of different rankings (e.g., results from different search engines on a given query) for the same set of data (e.g., pages relevant to the query), and the goal is to select a unique ranking that is as close as to the whole set of rankings according to a fully-defined criterion. In this example, the correspondence is that different search engines are the agents (voters) and each webpage matches an alternative, while the method, i.e., the algorithm for which the unique ranking is computed is the voting rule. It is obvious that in such applications, the decision on who is the winner of the election is not the only problem, since it is usually required a full ranking of the alternatives. Hence, the main question that voting theory tries to answer is which alternative best reflects the social good. In other words, given a set of agents who rank a number of alternatives, who is the one to be selected as a winner? The French mathematician, philosopher and one of the first political scientists, Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet, proposed the following intuitive criterion: a winner must be an alternative winning all the other alternatives in a pairwise comparison. This means that an alternative is preferred over any other alternative by the majority of the agents. For example, alternative a is pairwise preferred to b when more than half of the agents rank a higher in their preference compared to b. Unfortunately, the majority's preferences may be cyclic. For example, in an election with 3 alternatives, a may be preferred to b, b to c, and c to a. Then, the winner of the election can not be determined according to this criterion. This phenomenon is called *Condorcet paradox*, and in order to overcome it, many researchers have suggested choosing an alternative to be as close as possible to the Condorcet winner. There are different notions of proximity of how close an alternative is to the Condorcet winner and this leads to the consideration of different voting systems.

Nowadays, many organizations face complex management problems and want their decisions to be supported by a "scientific approach", which is called *decision analysis*. The analyst in charge of this preparation faces many diverse tasks: stakeholders identification, problem statement, elaboration of a list of possible actions, definition of one or several criteria for evaluating these actions, information gathering, sensitivity analysis, elaboration of a recommendation (for instance a ranking of the actions or a subset of "good" actions), etc. The desire or necessity to take multiple conflicting viewpoints into account for evaluating actions often makes this task even more difficult and in that case, we call it multi-criteria decision making (aiding). The expert must then try to synthesize the partial preferences (modeled by each criterion) into a global preference, on which a recommendation can be based. This is called preference aggregation. This aggregation problem is very similar and has been studied for a long time in the framework of voting theory. It consists of searching a "reasonable" mechanism, i.e., the voting system, aggregating the opinions expressed by several agents on the alternatives of an election, in order to determine a winner or to rank all alternatives in order of preference. Voting and in more general, social choice theory analyzes the links that exist (or should exist) between the individual preferences of the members of a society and the decisions made by this group when these decisions are supposed to reflect the collective preference of the group. The results obtained in social choice theory are valuable for multi-criteria decision making. There are indeed links between these two domains: it is easy to go from one to the other by replacing the words "action", "criterion", "partial preference" and "overall preference" by "alternative" (candidate), "agent" (voter), "individual preference" and "collective preference" [12].

The formal proximity between both problems does not imply that both problems are identical. In fact, multi-criteria decision making is a broader problem and social choice methods can serve as one of the research paths followed to solve the problem. In particular:

- The goal of a multi-criteria decision making process is not always to choose one action or a ranking of the actions. There are many other kinds of outcomes, unlike in social choice theory.

- Some conditions and properties look intuitive in social choice but are questionable in multi-criteria decision aiding, and conversely. Let us mention, for example, that anonymity is not relevant in multi-criteria decision making as soon as we wish to take criteria of different importance into account. Conversely, the set of potential actions to be evaluated is seldom given in multi-criteria decision aiding (contrary to the set of alternatives in social choice theory); it can evolve. The conditions telling us how an aggregation method should behave when this set changes (some actions are added or removed) are therefore more important in multi-criteria decision making than in social choice.

- The preferences to be aggregated in multi-criteria decision making are the outcome of a long modeling phase along each criterion. This modeling phase can sometimes lead to incomplete preferences, fuzzy preferences or preferences such that indifference is not transitive, while in voting settings the preferences are usually complete and transitive. In some circumstances, it is possible to finely model preference intensities or even to compare preference differences on different criteria [83]. Also, handling uncertainty, imprecision or indeterminacy is often necessary to arrive at a recommendation in multi-criteria decision making, contrary to social choice.

- In multi-criteria decision making, contrary to the classical social choice setting, it is not always necessary to completely construct the global preference. Indeed, it can occur that the decision maker can express their global preference with respect to some pairs of alternatives. For example, they are able to state that they prefer x to z and y to z but they hesitate between x and y. If they then use an aggregation method, it is in order to construct the preference only between x and y and not on the whole set of alternatives. While this is common in decision making it is only until recently that problems with partial preferences have started to be investigated in social choice literature [66, 84, 17].

Concluding, we have to state that social choice theory is one of the approaches that provides methods for solving problems that appear in collective Decision-making. In this post-doctoral research we are mostly interested for solving decision-making problems under this approach, i.e., a social choice theoretic perspective.

1.2 The basics regarding Argumentation Systems

Argumentation theory is the interdisciplinary study of the way conclusions can be derived through logical reasoning, i.e., claims based, soundly or not, on premises. It includes the arts and sciences of civil debate, dialogue, conversation, and persuasion. In addition, argumentation includes deliberation and negotiation which are concerned with collaborative decision-making procedures. More specific to computer science and artificial intelligence is the notion of an *argumentation framework*, or *argumentation system* (AF). An argumentation framework provides a way to deal with contentious information and draw conclusions from it. The idea behind argumentation framework is that reasoning can be performed by constructing and evaluating arguments, which are composed of a number of reasons for the validity of a claim. Arguments distinguish themselves from proofs by the fact that they are defeasible, that is, the validity of their conclusions can be disputed by other arguments. Whether a claim can be accepted therefore depends not only on the existence of an argument that supports this claim, but also on the existence of possible counter arguments, that can themselves be attacked by counter arguments.

Nowadays, most of the research on the topic of argumentation is based on the prominent work of Dung's on abstract argumentation framework [47]. The central concept in this work is that of an argumentation framework, which is essentially a directed graph in which the arguments are represented as nodes and the attack relations are represented by directed edges. Given such a graph, one can then examine the question of defining which set(s) of arguments can be accepted together: answering this question corresponds to defining an argumentation semantics. Various proposals have been formulated in this respect, and many semantics have been defined in the literature. For example, we can mention here the most fundamental ones that were proposed by Dung and are considered to be mainstream, since they share a basic property called admissibility and have been subject of much study, including the specification of proof procedures and of properties regarding computational complexity. Let us first define the basic notions. More formally, an argumentation framework is a pair $(\mathbf{A}, \mathbf{R})$, where $\mathbf{A}$ is a finite nonempty set of arguments and $\mathbf{R}$ is a binary relation on $\mathbf{A}$, called attack relation. Let $A, B \in \mathbf{A}$, $A\mathbf{R}B$ means that A attacks B. The sets of "accepted" arguments are called "extensions" and are determined according to the given semantics and whose definition is usually based on the following concepts. Let $A \in \mathbf{A}$ and $\mathcal{S} \subseteq \mathbf{A}$, $\mathcal{S}$ is *conflict-free* if and only if there does not exist $A, B \in \mathcal{S}$ such that $A\mathbf{R}B$. We say that $\mathcal{S}$ *defends* an argument A if and only if each attacker of A is attacked by an argument of $\mathcal{S}$ and $\mathcal{S}$ is an *admissible set* if and only if it is conflict-free and it defends all its elements. Dung proposed the following semantics: Let $\mathcal{E} \subseteq \mathbf{A}$. $\mathcal{E}$. We say that an extension is *complete* if and only if $\mathcal{E}$ is an admissible set and every argument which is defended by $\mathcal{E}$ belongs to $\mathcal{E}$. Also, we say that an extension is *preferred* if and only if $\mathcal{E}$ is a maximal admissible set (with respect to set inclusion $\subseteq$). The next one is the *grounded* extension which is a minimal (with respect to $\subseteq$) complete extension. Concluding, we say that an extension is *stable* if and only if $\mathcal{E}$ is conflict-free and attacks any argument $\mathcal{A} \notin \mathcal{E}$.

1.2.1 Argumentation for Decision Making

Decision making is often viewed as a form of reasoning toward actions and has been investigated by many different researchers including philosophers, economists, psychologists, and computer scientists for a long time. Any decision problem amounts to selecting the "best" action(s) that are feasible among different alternatives, given some available information about the current state of the world and the consequences of the potential actions. Available information can be incomplete or pervaded with uncertainty. Besides, the goodness of an action is judged by estimating how much its possible consequences fit the preferences of the decision maker. This agent is assumed to behave in a rational way, at least in the sense that her decisions should be as much as possible consistent with her preferences. Argumentation is a reasoning model based on the construction and the evaluation of interacting arguments. Those arguments are intended to support/explain/attack statements that can be decisions, opinions, etc. Argumentation has been used in decision making for different purposes like the following. First, as nonmonotonic reasoning (e.g. [47] where several frameworks have been developed for handling inconsistency in knowledge bases (e.g. [3, 5, 20]. Second, argumentation has also been extensively used for modeling different kinds of dialogues, in particular persuasion (e.g. [6]), and negotiation (e.g. [67]). Indeed, an argumentation based approach for negotiation has the advantage of exchanging, in addition to offers, reasons that support these offers. These reasons may lead their receivers to change their preferences. Consequently, an agreement may be more easily reached with such approaches, when in other approaches (where agent's preferences are fixed) negotiation may fail. Adopting such an approach in a decision problem has some obvious benefits. It is not only that the decision maker is provided with a "good" choice, but also with the reasons underlying this recommendation. Argumentation-based decision making is more akin with the way agents deliberate and finally make or also understand a decision/choice.

1.3 Post-doctoral's research contribution

In this post-doctoral research project we study preference aggregation problems that originate in social choice theory enriched with the consideration of reasoning expressed in the agents preferences when taking a decision. Therefore, we focus towards problems and applications lying on collective decision making whose solution needs to be based on aspects and computational techniques of social choice theory for aggregation of the preferences. We state that the contribution of this post-doctoral research in the field of collective decision making is significant since we consider notions of computational social choice in order to provide novel methods which follow an algorithmic approach for solving decision problems based on the aggregation of agents' preferences.

While classic collective decision making and preference aggregation methods deal with agents' unjustified preferences for actions, we go one step further by studying the case where the collective decision needs to be based on the agents' preferences and the reasoning provided for these preferences. Our research motivation is that a decision made by agents who understand the reasoning of the collective preference has better chances to be accepted and used in practice because the outcome is a rational process that can be justified. We propose two algorithmic collective decision making methods in order to aggregate agents' preferences. Both are based on argumentation framework and social choice theory. The first one is a novel qualitative decision process and the second is a rather quantitative one. In the first approach we build a scheme with a framework made out of the preferences of the agents which is called *ranking-completion* argumentation framework. This framework follows a qualitative approach on the agents' preferences by taking into account only once the different preferences. The preference profile computed by the coherent sets of preferences is "justified" since it is the outcome of the argumentation framework. The justified preferences can then be the input for a social choice method where the aggregation of them will give the decision. We show that the proposed scheme build "justified" profiles that can overcome some of the social choice deficiencies. Our second approach follows a quantitative scheme for quantifying the deliberation phase by defining a new *voting* argumentation framework which is build from the preferences of the agents. Here, the preferences of the agents are depicted into the framework as many times as they appear in the original profile. We define a new class of acceptability semantics to compute the set of coherent preferences which construct a new "justified" preference profile. For this scheme, we prove some theoretical results regarding well-known properties that originate in argumentation and social choice theory, and the conditions that are needed so that they can be satisfied.

The next problem, we deal with in this post-doctoral research, is the one of collective decision making under multiple criteria. Here, the agents form the preferences on a cardinal basis, in contrast to ordinal preferences as in the abovementioned problems. In addition, the agents' preferences are expressed over a set of multiple criteria and the goal is also to reach an acceptable collective decision aggregating these preferences. We provide a novel modelling of the multi-criteria decision making problem as an inconsistent knowledge base, and we explain how to benefit from the reasoning capabilities of existential rules. The repairs of this knowledge base which represent the maximally consistent point of views can be used for computing the collective decision.

In order to fulfill our research we turn our attention in real decision making applications where the need of implementation of methods and algorithms of social choice is at high priority. Therefore, we design a decision-making software tool that applies in decision problems of agricultural engineering for aggregating agents' preferences. The orientation of our implementation is to provide the decision makers a tool that can be directly applied to real problems and hence, our tool is implemented taking into account Information Visualization techniques to provide user-friendliness. The tool uses methods of computational social choice and argumentation for collective decision making and hence, it is composed of two main systems, i.e., the social choice system and the deliberation system. We mainly focus on the social choice system and how it can be applied to aggregate agents' preferences over different alternatives having as an objective the valorization of materials, which is a well-known and crucial decision problem in agricultural engineering. We complete the picture by providing the implementation of algorithms that were used to support the theoretical results of this research. The basic functionality of these algorithms is twofold. First, they include several methods for the computation of the extensions of the argumentation frameworks that are build from agents preferences. Second, they build several social choice methods for computing an aggregation of the preference profile. Hence, the implemented algorithms will be the foundational basis on which we are going to build future software tools for collective decision making by using reasoning based methods.

Chapter 2

Collective decision making mechanisms based on the reasoning of the agents' preferences

In this chapter we study the problem of collective decision making among a set of alternative choices when agents provide their preferences and the reasoning behind their choices. One of the most popular approach for solving the problem is by using methods and techniques of social choice and Voting theory. However, classic voting methods take the agents preferences as is, i.e., de facto, which means that they do not investigate on the reasoning behind them. To this end, we go one step further and explore how the agents' rationale can be formulated inside the classical voting setting in order to justify their choices. We believe that a decision made by decision makers where the reasoning of the collective decision is clear and solid has better chances to be accepted and used in practice because a justified outcome is a rational process that can be justified by the arguments. Therefore, we propose two novel decision-making algorithmic procedures based on argumentation and social choice (for preference aggregation) which permits us to explore the effect of reasoning and deliberation along with voting for the decision process. The first method is a qualitative one which builds an argumentation framework from the agents' preferences and focuses on some of the social choice deficiencies. This method is presented [24] where we also show that justifications on the preferences can help us overcome some of these deficiencies. This work is published in the journal "Progress in Artificial Intelligence". The second approach is a quantitative method where the goal is to quantify the reasoning phase by defining a new voting argumentation framework build from the agents' preferences and its acceptability semantics. We prove for this approach theoretical results regarding well-known properties from argumentation and social choice theory, and the conditions that are needed so that they can be satisfied. The work following this procedure has been presented in the "International Conference on Agents and Artificial Intelligence (ICAART '19)" [62].

2.1 A Decision-Making approach where Argumentation added value tackles Social Choice deficiencies

Abstract

Collective decision-making in multi-agents systems is classically performed by employing social choice theory methods. Each member of the group (i.e., agent) expresses preferences as a (total) order over a given set of alternatives, and the group's aggregated preference is computed using a voting rule. Nevertheless, classic social choice methods do not take into account the rationale behind agents' preferences. Our research hypothesis is that a decision made by a group of participants understanding the qualitative rationale (expressed by arguments) behind each other's preferences has better chances to be accepted and used in practice. Accordingly, in this work, we propose a novel qualitative procedure which combines argumentation with computational social choice for modeling the collective decision-making problem. We show that this qualitative approach produces structured preferences that can overcome major deficiencies that appear in the social choice literature and affect most of the major voting rules. Hence, in this paper we deal with the Condorcet Paradox and the properties of Monotonicity and Homogeneity which are unsatisfiable by many voting rules.

2.1.1 Introduction

Taking collective decisions is a part of our everyday life. From the simplest ones, e.g., choosing which movie we are going to watch in the theater, to the most complex ones, e.g., selecting a government, a collective decision has to be made. The way to achieve a decision that satisfies the group members though can be a very complex task. It is plausible to wonder what is the rationale behind a decision in addition to the decision itself. Usually the involved participants (decision makers) take their decisions based on their preferences which can be expressed by different viewpoints, criteria and aspects that they consider to be important. One should wonder then what happens when we want to take a justified group's decision where the reasoning of the preferences is clear and the decision should be as fair as possible. This leads us to the following questions.

- *How do agents form their thoughts and justify their preferences?*
- *How should we aggregate them in order to have a "democratic" collective decision?*

That are the problems we are dealing with in this paper.

The commonly used way of making a collective decision is using social choice theory and aggregation methods. Each agent of the group expresses her preference as a total order over a set of alternatives, and then the group's preference is computed from the individual preferences using a voting rule. The original motivation of social choice theorists was to model, analyse and give solutions to political decision making in groups of people, but nowadays its basic principles are used in modelling and analysing the kinds of interaction taking place in multi-agent systems. In classical voting the collective decision is computed from quantitative methods by taking into account only the agents' preferences without knowing why the agents have these preferences and what is the rationality behind it. We refer the reader to the Handbook of Computational Social Choice [33] for an analytical description of the problem and the classical voting methods used in the literature. Thus, classical social choice presents a framework where the justifications for the agents' preferences are not considered.

In order to tackle the previously mentioned questions, we believe that decision support systems based on qualitative methods, where the agents understand the rationale behind preferences, have better chances to be accepted by the decision makers. This gives us the motivation to propose a qualitative decision-aiding process for multi-agent systems which combines argumentation [21, 47] with computational social choice [33]. We believe that enriching the collective decision-making procedure with an argumentation framework can benefit the procedure in a twofold way. First, given that agents present justifications for their preferences, it can provide the reasoning behind the decision and secondly, it can model the deliberation phase prior to voting for taking a group decision. Modelling the deliberation phase when agents form their preferences with an argumentation framework permits us to compute extensions, i.e, the collective viewpoint of the group, which construct a preference profile that is "justified". Hence, the justified preference profile can be seen as a type of structured profile which is the outcome of a pre-voting phase that consists of a deliberation procedure. Consequently, this gives us the motivation to study in this paper a decision-making approach based on argumentation, but also on social choice which provides the means to aggregate the structured outcome, i.e., the justified preference profile.

In our approach we are going to place the decision problem within the boundaries of an abstract argumentation framework. Abstract argumentation theory is easy to understand and provides a robust tool for non-monotonic reasoning. It was first introduced by Dung in 1995 [47] and is based on the construction, the exchange and the evaluation of interacting arguments. The argumentation systems are modelled by graphs, where the nodes represent the arguments, and the edges represent the attacks between them. Various semantics defined by Dung and other researchers have been proposed to identify the acceptability of sets of arguments, which are based on the attack relations between them.

In the problem considered in this paper, the decision to be recommended lies on a set of alternative options, which will be referred in the reminder of the paper as *alternatives*. The decision will be derived from the justified preferences of the set of *agents* over the set of the alternatives. The justified preferences are the outcome of a debate phase (deliberation) where each agent reveals her preferences by providing a ranking over the alternatives and a justification for this ranking. The collection of agents' rankings is known as *preference profile*. The preference profile of the agents along with the justifications are used to build the arguments and then the argumentation framework will help us build the *justified preference profile* which includes the preferences produced after the deliberation and corresponds to the different collective viewpoints of the agents. The objective is to fairly aggregate the justified viewpoints of the agents and hence, the justified preference profile is reported to a *voting rule*, which then singles out the winning alternative and the ranking of the remaining ones.

The classical problem in social choice theory is which voting rule is the most appropriate for aggregating the preference profile. Unfortunately, due to two fundamental impossibility results from social choice theory there is no hope of finding a voting rule that can be "perfect". The first one was imposed by Arrow in 1950 [13] and the second one by Gibbard in 1973 and Satterthwaite in 1975 [58, 81]. Due to these results we know some vital criteria cannot be satisfied all at the same time. Despite that, social choice theory has enhanced our perception among proposed voting rules, where each of them has different characteristics, qualities, and weaknesses but all of them have the same goal, i.e., to elect the fairest socially outcome. It should be noted here that each rule has some assumptions of what is the fairest outcome. For example, when there are two alternatives and an odd number of agents, the majority rule is unanimously considered a perfect, in terms of fairness, method of selecting the winner. However, when we have three alternatives or more, majority rule is not appropriate anymore and another rule should be used. One of the most prominent rules in the history of social choice which is generally acclaimed as a founding method of the field is the one proposed by the Marquis de Condorcet in 1785 [40], and bears his name. The *Condorcet method* relies on comparisons between each pair of alternatives and the winner of the election, which is known as the *Condorcet winner*, reflects the best choice for the social good according to social choice theorists. An alternative x is said to beat alternative y in a *pairwise election comparison)* if a majority of agents prefer x to y, i.e., rank x above y. The alternative who beats every other alternative in a pairwise comparison is the winner of the election. However, the Condorcet method has a major weakness as there are preference profiles where the Condorcet winner does not exist. This problem arises when the preferences of the majority are cyclic, i.e., not transitive. For example, if we have three alternatives A, B, C and the results of the pairwise comparisons are: A beats B, B beats C and C beats A, then we say that a *voting cycle* occurs. This contradictory phenomenon is known as the *Condorcet paradox* as defined by Black in 1958 [26]. Despite this paradox, the Condorcet criterion is widely acclaimed as one of the most intuitive ways of voting and will be used to aggregate the justified preferences. To strengthen this perspective, we show that our method always provides justified preference profiles where the Condorcet paradox does not occur.

In order to evaluate the quality of well-known voting rules social choice theorists have defined properties that should be satisfied from a social point of view. Unfortunately, there are well-known voting rules that fail to satisfy major "socially desirable" properties. We can mention here, for example, Dodgson's voting rule who fails to satisfy the properties of monotonicity and homogeneity [55, 31]. These properties are considered by social choice theorists to be extremely basic; a voting rule is *monotonic* if it is indifferent to pushing a winning alternative upwards in the preferences of the agents, and *homogeneous* if it is invariant under duplication of the electorate. Apart from Dodgson's rule, other important rules, i.e., Plurality with Runoff, Alternative Vote (Instant Runoff Voting), Nanson's and Coombs', also fail to satisfy monotonicity [54].

Our work Our research is driven from use-cases where the objective is to reach a unanimous or close to a consensus decision among the group (group consensus approach). It is common in this approach that a deliberation phase is conducted among the agents in the group in order to express their preferences and the reasoning (justifications) behind them. We model this group decision-making problem with the following proposed method where the framework built from the justifications of the preferences, can lead to extensions where we have a reduced number of ambiguous preferences among the agents.

More specifically, in our technique, we will use the agents' preferences (rankings) and the justifications for those rankings to compute arguments which will be the base of the argumentation framework. These arguments will rely on the justification of the pairwise individual preferences given by each agent. The set of coherent viewpoints, i.e., the set of extensions $\boldsymbol{E}$, will be computed according to the "preferred" semantics and each extension corresponds to a ranking of the alternatives which contains information about the different viewpoints of the agents' preferences. Hence, the collection of all the extensions provides the justified preference profile. When there is no consensus among the agents' preferences, various possible extensions exist. Hence, we cannot take a decision based only on the argumentation framework. That leads to the consideration of a classical voting problem which has as input the justified preferences of the agents over the alternatives. The concluding voting problem gives us the motivation to study the following social choice theoretic properties, i.e., the Condorcet paradox, monotonicity and homogeneity. In a nutshell our contribution is as follows. *Given the preference profile we show that the construction of the justified preference profile permits a type of structured preferences where the Condorcet paradox can be avoided, as well as other social choice properties can be satisfied. Hence, we can use the Condorcet method to obtain the final ranking.*

Discussion The intuition behind the aggregation of the rankings extracted from extensions and the reason for considering them as (virtual) agents, i.e., the justified preference profile, can be seen from both an argumentative and social choice perspective. From an argumentative perspective, an extension can be seen as a consistent possible interpretation of justified preference relations and a subset of the group's collective viewpoint. Hence, the unique ranking argument entailed in an extension can be considered then as a part of the collective decision and hence, the corresponding ranking can be considered as a new individual agent. Therefore, in order to compute a collective decision taking into account all the possible viewpoints of the group, we must apply an aggregation mechanism/voting function on the (virtual) agents extracted from the extensions' rankings. From a social choice perspective, the "justified" votes are related to original preference profile. It is the justifications built in the arguments that define and compute (according to used semantics) the justified preference profile. By using "preferred" semantics in our approach we show that the Condorcet paradox can be avoided and is let for future work to see if other kind of semantics, e.g., adding weights to extensions with regards to the importance of the number of agents supporting the preference relations, can lead to the same results. Concluding, given the two abovementioned intuitive perspectives, we state that our approach leads to an interesting voting problem. It is therefore meaningful to study social choice properties for the input's profile, which is the justified preference profile. Hence, the need to prove the theoretic results included in this paper.

Related work To the best of our knowledge, the application of argumentation into social choice theory with the objective of aiding the decision-making under the social choice perspective is a relatively new domain, however, several work has been done on the combination of each pair of these fields. Decision-making has begun evolving from the 60s when Bernard Roy in 1966 [19] and in 1968 [78] introduced the class of ELECTRE methods for aggregating preferences expressed on multiple criteria, which set the foundations of the "outranking methods" that were further deployed by Ostanello in 1985 [73] and Roy in 1991 [79]. Decision-making and social choice theory are two closely correlated fields whose objective is to aggregate the partial preferences into a collective preference. Arrow and Raynaud in 1986 [12] were the first that presented a general exploration of the links between social choice theory and decision-making. Social choice theory has been integrated in the analysis of some popular aggregation methods in multi-criteria decision-making. Let us mention, for example, the ordinal methods in multi-criteria decision-making which were developed by Roy [79] and Roy and Bouyssou [80] and are based on the Condorcet method. In addition, scoring voting methods like the Borda count are integrated in the cardinal methods for multi-criteria aggregation, e.g., Keeney and H. Raiffa [64] and Von Winterfeldt and Edwards [83].

Several researchers have proposed the use of argumentation in decision-making. The work of Fox and Parsons in 1997 [56] is one of the first works that tried to deploy an argumentative approach to decision-making stating the difference between argumentation for actions and argumentation for beliefs. Most of the argumentation-based approaches objective is to select the best solution (alternative option), e.g., Karacapilidis and Papadias [60] and Morge and Mancarella [72]. On the contrary, in decision-making several different problem statements with different objectives are allowed, i.e., choosing, rejecting, ranking, classifying the set of alternatives. Regarding the aggregation, several approaches like the ones by Amgoud et al. [4] and by Bonnefon and Fargier [28] used procedures based only on the number or the strength of arguments, while in decision-making there have been proposed many aggregation procedures. Another example of work towards this direction is the one by Amgoud and Prade [7] which proposes an abstract argumentation-based framework for decision-making. The model follows a 2-step procedure where at first the arguments for beliefs and options are built and at the second step we have pairwise comparisons of the options using decision principles.

There have been also studied many problems on the intersection of social choice and argumentation which are loosely related to our work. Most of the issues in this research area view the problem from an argumentative perspective and deal with collective argumentation. Most of the works refer to the problem of aggregating individual argumentation frameworks to a collective one. The aggregation mechanisms provided to solve the problem rely on social choice, which provides the means to accomplish that. An informative survey on these problems in collective argumentation is provided by Bodanza et al. [27]. This area of research in collective argumentation is not only restricted to finding aggregation mechanisms. For example, in the work of Airiau et al. [2] the goal differentiates and instead they study the computational complexity of a problem defined as the "rationalisability problem". In this problem each agent has its own argumentation framework AF_i and the aim is to identify if there are a single master argumentation framework AF, an association of arguments with values and a profile of preference orders over values that can "explain" all the AF_i (i.e, AF rationalizes the set of AF_i).

There are a number of ways for formalizing the concept of reasoning. Hence, the reasoning behind agents' preferences has also been investigated from a different perspective than we do. One prominent work on identifying the reasons in preferences is the one by Dietrich and List [46], where they formulate a reason-based theory of rational choice in which agents' form preferences according to their motivating reasons. In this work they also clarify the relationship between deliberation for reasons and for rational choices.

2.1.2 Preliminaries

In the following, we present several notions needed to go further with our approach.

Social Choice Theory

We consider a set of $N = \{1, \ldots, n\}$ *agents* and a set of *alternatives* A, $|A| = m$. Each agent $i \in N$ has preference relations ($\succ$) over the alternatives denoted with $x \succ_i y$ which means that agent i *prefers* alternative x to y. We define that each irreflexive preference relation satisfies transitivity antisymmetry and comparability, and hence, the set of all the preference relations for agent i produces a linear (strict total) order $\succ_i$ on A, i.e., the ranking of agent i over the alternatives. Let $\mathcal{L}_A$ be the set of linear orders over A. A *preference profile* $\succ_{PP} = \langle \succ_1, \ldots, \succ_n$

$\rangle \in \mathcal{L}_A^n$ is a collection of the linear orders for all the agents. For each fixed value of n, a *voting rule* is a mapping $f : \mathcal{L}_A^n \rightarrow 2^A \setminus \{\emptyset\}$ from preference profiles to nonempty subsets of alternatives, which designates the winner(s) of the election. For two alternatives $x, y \in A$ and $\succ_{PP} \in \mathcal{L}_A^n$, alternative x *beats* y in a *pairwise comparison* if $|\{i \in N : \; x \succ_i y\}| > n/2$, that is, if a (strict) majority of agents prefer x to y. The winner according to the *Condorcet method* is an alternative that beats every other alternative in a pairwise comparison. A *Condorcet winner* does not always exist due to the following paradox. The *Condorcet paradox* (also known as voting paradox or the paradox of voting) is a situation in which the collective preference profile can be cyclic (i.e., not transitive), even if the preferences of individual agents are not cyclic. A *voting cycle* occurs when we have three alternatives x, y, z such that x beats y, y beats z and z beats x in pairwise comparisons.

Argumentation

In order to be general with regards to the deliberation step, we are using the abstract argumentation framework proposed by Dung in 1995 [47]:

Definition 1 (Argumentation framework)**.** *An argumentation framework (AF) is a pair* $(\mathbf{A}, \mathbf{R})$*, where* $\mathbf{A}$ *is a finite nonempty set of arguments and* $\mathbf{R}$ *is a binary relation on* $\mathbf{A}$*, called attack relation. Let* $\mathcal{A}, \mathcal{B} \in \mathbf{A}$*,* $\mathcal{A}\mathbf{R}\mathcal{B}$ *means that* $\mathcal{A}$ *attacks* $\mathcal{B}$*.*

The coherent sets of arguments (called "extensions") are determined according to a given semantics whose definition is usually based on the following concepts:

Definition 2 (Conflict-free set, defense and admissibility)**.** *Given an AF* $(\mathbf{A}, \mathbf{R})$*, let* $\mathcal{A}, \mathcal{B} \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$*. We say that*

- $\mathcal{S}$ *is* conflict-free *if and only if there do not exist* $\mathcal{A}, \mathcal{B} \in \mathcal{S}$ *such that* $\mathcal{A}\mathbf{R}\mathcal{B}$*,*
- $\mathcal{S}$ defends *an argument* $\mathcal{A}$ *if and only if each argument that attacks* $\mathcal{A}$ *is attacked by an argument of* $\mathcal{S}$*, and*
- $\mathcal{S}$ *is an* admissible set *if and only if it is conflict-free and it defends all its elements.*

Definition 3 (Semantics)**.** *Given an AF* $(\mathbf{A}, \mathbf{R})$*, let* $\mathcal{E} \subseteq \mathbf{A}$*.* $\mathcal{E}$ *is*

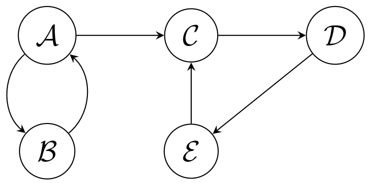
- *a* complete *extension if and only if* $\mathcal{E}$ *is an admissible set and every argument which is defended by* $\mathcal{E}$ *belongs to* $\mathcal{E}$*,*
- *a* preferred *extension if and only if* $\mathcal{E}$ *is a maximal admissible set (with respect to set inclusion),*

- *the* grounded *extension if and only if $\mathcal{E}$ is a minimal (with respect to set inclusion) complete extension, and*

- *a* stable *extension if and only if $\mathcal{E}$ is conflict-free and attacks any argument $\mathcal{A} \notin \mathcal{E}$.*

Given a semantics s, the set of extensions of $(\mathbf{A}, \mathbf{R})$ *is denoted by* $\boldsymbol{E}_s$.

Example 1. *Given an AF* $(\mathbf{A}, \mathbf{R})$ *with* $\mathbf{A} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$ *and* $\mathbf{R} = \{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{A}), (\mathcal{A}, \mathcal{C}), (\mathcal{C}, \mathcal{D}), (\mathcal{D}, \mathcal{E}), (\mathcal{E}, \mathcal{C})\}$



- *The complete extensions are* $\{\}$, $\{\mathcal{A}, \mathcal{D}\}$ *and* $\{\mathcal{B}\}$

- *The preferred extensions are* $\{\mathcal{A}, \mathcal{D}\}$ *and* $\{\mathcal{B}\}$

- *The unique grounded extension is* $\{\}$

- *The stable extension is* $\{\mathcal{A}, \mathcal{D}\}$

It should be noted that in this paper we want to ensure that each extension represents a full ranking over the alternatives justified by preference relation arguments, which is needed since extensions will be used as (virtual) agents. In order to do that, we focus on the preferred semantics since it ensures to cover all the maximal sets of arguments, which corresponds to the "viewpoints" on the possible rankings of alternatives. Please note that future work will consider the study of other argumentation semantics.

Based on these notions, we can now present the model combining the strengths of social choice and argumentation.

2.1.3 A decision model based on justified preferences

In the proposed model we are considering the case of taking a collective decision using a qualitative argumentative approach and voting theory, in order to reflect real-life decision problems where a deliberation phase is present. In our problem the input is a set of alternatives as well as the justified preferences of agents over these alternatives. In this paper, each agent provides a justification for each of her preference relations on the alternatives and the preferences are restricted to satisfy a transitive relation so as to allow for the ranking of the alternatives to be built. Observe that the suggested process is an argumentative approach that relies on combining the "qualitative" preferences, which is incomparable to a voting rule whose role is to aggregate the individual preferences with quantitative methods. We use this information to formulate arguments which express the agents' preferences. More precisely, we are going to distinguish between three types of arguments: *"preference relation"* arguments, *"ranking"* arguments and *"generic"* arguments.

Preference relation arguments. A preference relation argument $\mathcal{A}_{xy}$ represents a *justification* given by an agent to consider the preference $x \succ y$. Note that we may have multiple $\mathcal{A}_{xy}$ arguments, in the case where some agents have different justifications for the preference $x \succ y$. The set of preference relation arguments is denoted $\mathbf{A}_{PR}$.

It should be noted that due to what they represent, the arguments $\mathcal{A}_{xy}$ and $\mathcal{A}_{yx}$ cannot be considered together in a coherent view point since they are "opposed". Consequently, we assume that those arguments are attacking each other.

Example 2. *Consider a set of three agents $\{1, 2, 3\}$ deciding which movie they want to watch at the theater tonight. A collective decision has to be made out of the three movies that are played in the nearest theater, i.e.,* "Beauty and the beast", "Free fire", "Going in style". *We denote the set of the alternatives $\{a, b, c\}$. The preference relations of the agents over $\{a, b, c\}$ and their justifications are the following:*

- $\mathcal{A}_{ab}$: $a \succ_1 b$, *because a has IMDB rating 7.7 and b has 7.2*
- $\mathcal{A}_{bc}$: $b \succ_1 c$, *because b has IMDB rating 7.2 and c has 6.8*
- $\mathcal{A}_{ac}$: $a \succ_1 c$, *because a has IMDB rating 7.7 and c has 6.8*
- $\mathcal{A}_{ba}$: $b \succ_2 a$, *because b has a higher rating than a in Rotten Tomatoes*
- $\mathcal{A}'_{bc}$: $b \succ_2 c$, *because b has a higher rating than c in Rotten Tomatoes*
- $\mathcal{A}_{ca}$: $c \succ_2 a$, *because c has a higher rating than a in Rotten Tomatoes*
- $\mathcal{A}'_{ab}$: $a \succ_3 b$, *because Agent 3 prefers fantasy to action movies*
- $\mathcal{A}_{cb}$: $c \succ_3 b$, *because Agent 3 dislikes action movies*
- $\mathcal{A}'_{ca}$: $c \succ_3 a$, *because Agent 3 prefers horror to fantasy movies*

Note that the preferences of the agents are cyclic and trigger the Condorcet paradox: there is a majority of agents for $a \succ b$, another majority for $b \succ c$ and a third majority for $c \succ a$.

We can represent the above arguments in the following graph which is depicted in figure 2.1:

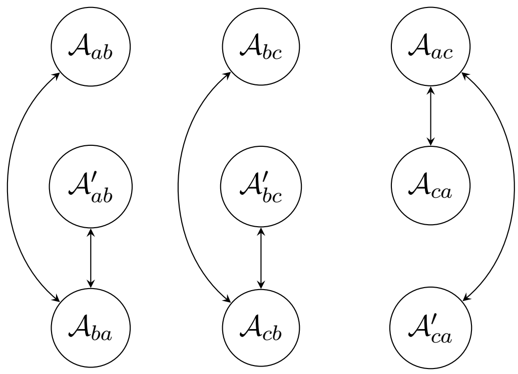


Figure 2.1: The argumentation graph containing the preference relation arguments and their attacks.

Ranking arguments. A ranking argument represents one of the possible rankings over the considered alternatives. It is important to note that in our setting, we always consider all the possible ranking arguments; it will be the agents' prerogative to justify why a ranking should not be considered as we will see below.

We denote by $\mathbf{A}_{\mathcal{R}}$ the set of all the possible ranking arguments and by $\mathbf{A}_{\mathcal{R}_{x\cdots y}}$ the set of ranking arguments where the preference $x \succ \cdots \succ y$ is satisfied. Moreover, we define a special ranking argument $\mathcal{A}_B$ that represents a ranking without preference; it can be seen as the blank vote resulting from either non-transitive preference relations or no justified preferences.

Like preference relation arguments, we consider ranking arguments as mutually inconsistent. For this reason, we assume that ranking arguments are attacking each other, with the exception of $\mathcal{A}_B$ that attacks no argument. In this way, we represent the fact that having a reason to consider a ranking forbids the possibility of considering blank voting.

Furthermore, ranking arguments can be attacked by preference relation arguments. Indeed, giving a justification for $x \succ y$ (i.e., enunciating an argument $\mathcal{A}_{xy}$) is a reason for not considering all the rankings with $y \succ x$ (i.e., $\mathbf{A}_{\mathcal{R}_{yx}}$); here, $\mathcal{A}_{xy}$ is attacking the elements of $\mathbf{A}_{\mathcal{R}_{yx}}$. Please note that we want the ranking arguments to be justified by preference relation arguments, hence, we do not allow ranking arguments to attack conflicting preference relation arguments. If we allowed this to happen, then the ranking arguments would be able to defend and justify themselves directly (in the context of Dung's abstract argumentation), which is not desirable.

Example 2 (cont.)**.** *We complete the previous argumentation framework with the set of ranking arguments $\mathbf{A}_{\mathcal{R}}$ for all the possible permutations of $\{a, b, c\}$ while adding the attacks between the preference relation arguments and the ranking arguments. We obtain:*

- $\mathcal{A}_{ab}$ *and* $\mathcal{A}'_{ab}$ *attack* $\mathcal{A}_{\mathcal{R}_{bac}}$, $\mathcal{A}_{\mathcal{R}_{bca}}$ *and* $\mathcal{A}_{\mathcal{R}_{cba}}$

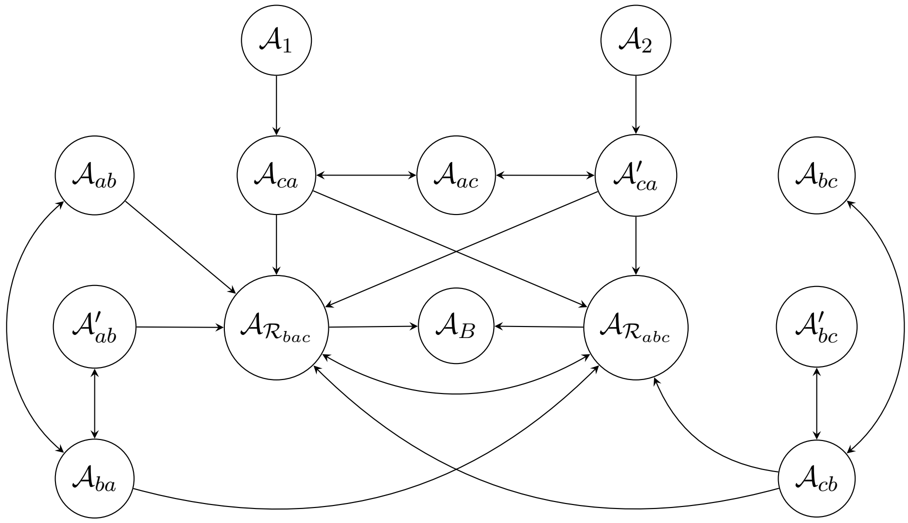


Figure 2.2: A partial presentation of the argumentation graph for two particular ranking arguments.

- $\mathcal{A}_{ba}$ *attacks* $\mathcal{A}_{\mathcal{R}_{abc}}$, $\mathcal{A}_{\mathcal{R}_{acb}}$ *and* $\mathcal{A}_{\mathcal{R}_{cab}}$
- $\mathcal{A}_{bc}$ *and* $\mathcal{A}'_{bc}$ *attack* $\mathcal{A}_{\mathcal{R}_{cab}}$, $\mathcal{A}_{\mathcal{R}_{acb}}$ *and* $\mathcal{A}_{\mathcal{R}_{cba}}$
- $\mathcal{A}_{cb}$ *attacks* $\mathcal{A}_{\mathcal{R}_{bca}}$, $\mathcal{A}_{\mathcal{R}_{bac}}$ *and* $\mathcal{A}_{\mathcal{R}_{abc}}$
- $\mathcal{A}_{ac}$ *attacks* $\mathcal{A}_{\mathcal{R}_{bca}}$, $\mathcal{A}_{\mathcal{R}_{cba}}$ *and* $\mathcal{A}_{\mathcal{R}_{cab}}$
- $\mathcal{A}_{ca}$ *and* $\mathcal{A}'_{ca}$ *attack* $\mathcal{A}_{\mathcal{R}_{abc}}$, $\mathcal{A}_{\mathcal{R}_{bac}}$ *and* $\mathcal{A}_{\mathcal{R}_{acb}}$

Generic arguments. Generic arguments regroup all the other possible arguments that can arise during a debate. In particular, those arguments are only able to attack other generic arguments and preference relation arguments (for instance if the reason given for considering $x \succ y$ is itself justified to be wrong).

Example 2 (cont.)**.** *During the debate phase new arguments appear. Agent 1 contradicts the information of Agent 2 about alternative c because c has lower rating in Rotten Tomatoes and also contradicts Agent 3 because c is a comedy movie. Hence, we form the following arguments* $\mathcal{A}_1$ =*(c has lower rating than a in rotten tomatoes) and* $\mathcal{A}_2$ = (*c is comedy*) *such that* $\mathcal{A}_1$ *attacks* $\mathcal{A}_{ca}$ *and* $\mathcal{A}_2$ *attacks* $\mathcal{A}'_{ca}$*.*

The following Figure 2.2 gives a partial representation of the new arguments and attacks.[1]

[1]Please note that for the sake of clarity, we are not drawing all the edges in the argumentation graph, but a subset of the edges demonstrating the attacks between preference relation arguments and ranking arguments.

It is important to note that while the flexibility offered by the abstract argumentation setting is convenient for its generality, it can also lead to undesirable behaviors. Hence, we propose the following restriction.

Axiom 1 (Independence of preference justifications)**.** *Given two preference relation arguments $\mathcal{A}_{xy}$ and $\mathcal{A}_{uv}$, such that $\{x, y\} \neq \{u, v\}$, there is no generic argument $\mathcal{A}_g$ such that both paths of attacks from $\mathcal{A}_g$ to $\mathcal{A}_{xy}$ and from $\mathcal{A}_g$ to $\mathcal{A}_{uv}$ exist.*

The intuition is that the discussions about each pairwise preference are independent, i.e., a generic argument cannot have an impact on preferences over different alternatives. We assume that any generic argument general enough to have an effect on several pairwise comparisons can be separated into "pairwise exclusive" arguments. For instance, an argument rebutting the facts that x is over y and u is over v because of some reason R can be separated into two generic arguments: one attacking $\mathcal{A}_{xy}$ and one attacking $\mathcal{A}_{uv}$ (both because of reason R).

Computing the justified profile. Using the arguments and attacks shown above in an abstract argumentation framework, called the *ranking-completion argumentation framework*, it is possible to compute the sets of "coherent preferences", represented by the extensions.[2] Hence, it is important to remark that this process allows us to move from the direct aggregation of agents' preferences to the aggregation of rational and justified sets of preferences (and their corresponding rankings).

More precisely, multiple extensions are computed (unless all the arguments coincide, i.e, having a consensus among the agents' preferences). Each extension contains the preference relation arguments and a single ranking argument which corresponds to a coherent aggregation of possible preference relations with their justifications. Hence, it is now possible to consider the extensions as (virtual) agents and aggregate their rankings. Given a semantics s, the set of justified preferences is denoted by $\mathcal{JP}^s$; hence, $|\mathcal{JP}^s| = |\boldsymbol{E}_s \setminus \{\mathcal{E} \in \boldsymbol{E}_s : \mathcal{A}_B \in \mathcal{E}\}|$, where $\boldsymbol{E}_s$ is the set of extensions obtained thanks to semantics s. We consider the ranking of each extension (except if the extension contains the blank vote) as a *justified preference* $\mathcal{JP}_k$, with $k \in [1, |\mathcal{JP}^s|]$. Each justified preference has preferences over the alternatives denoted with $x \succ_{\mathcal{JP}_k} y$ which means that justified preference $\mathcal{JP}_k$ *prefers* alternative x to y.

Informally, the collective justified preference profile is the set of all the justified preferences.

Definition 4 (Justified pref. profile)**.** *A justified preference profile $\succ_{\mathcal{JP}^s} = \langle \succ_{\mathcal{JP}_1}, \ldots, \succ_{\mathcal{JP}_{|\mathcal{JP}^s|}} \rangle \in \mathcal{L}_A^{|\mathcal{JP}^s|}$ is a collection of linear orders for all the justified preferences obtained thanks to a semantics s.*

[2] Please note that we assume that no odd-length attack cycle may exist between generic arguments in the argumentation framework (such cases would be handled during the actual deliberation). Indeed, allowing the existence of odd-length cycles could lead to the computation of an empty extension which is not a coherent preference, since it is the result of an ambiguous deliberation and no ranking argument would be justified.

For instance, using the preferred semantics would produce a justified preference profile $\succ_{\mathcal{JP}^p}$

Example 2 (cont.)**.** *We use the "preferred" acceptability semantics in order to compute the extensions and thus, the justified preference profile* $\succ_{\mathcal{JP}^p}$*. The set of extensions is*

$$\boldsymbol{E} = \left\{ \begin{array}{l} \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_{ab}, \mathcal{A}'_{ab}, \mathcal{A}_{bc}, \mathcal{A}'_{bc}, \mathcal{A}_{ac}, \mathcal{A}_{\mathcal{R}_{abc}}\}, \\ \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_{ba}, \mathcal{A}_{bc}, \mathcal{A}'_{bc}, \mathcal{A}_{ac}, \mathcal{A}_{\mathcal{R}_{bac}}\}, \\ \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_{ab}, \mathcal{A}'_{ab}, \mathcal{A}_{cb}, \mathcal{A}_{ac}, \mathcal{A}_{\mathcal{R}_{acb}}\}, \\ \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_{ba}, \mathcal{A}_{cb}, \mathcal{A}_{ac}, \mathcal{A}_B\} \end{array} \right\}$$

We derive the justified preference profile

$\succ_{\mathcal{JP}^p} = \langle a \succ_{\mathcal{JP}_1} b \succ_{\mathcal{JP}_1} c, b \succ_{\mathcal{JP}_2} a \succ_{\mathcal{JP}_2} c, a \succ_{\mathcal{JP}_3} c \succ_{\mathcal{JP}_3} b \rangle$.

The aggregation of the justified preference profile gives a *as the Condorcet winner, while the original preference profile leaded to the voting paradox.*

As noted before, the justified preference profile can have multiple justified preferences (extensions), so we refer to classical social theory for aggregating them and hence, the collective decision produced by our method is a ranking of the alternatives. As we are going to see in the next section, the construction of the justified preference profile allows to avoid the voting paradox when the Condorcet method is used to declare the winner.

There can be cases where the justified preference profile consists only of blank votes and hence, a full tie between the alternatives may exist. This may happen if the reasoning part does not affect the preference profile, i.e., the generic arguments are not present or mutually attack each other. Therefore, this phenomenon shows the importance of deliberation and reasoning on the preferences and how a structured justified profile can behave in a better way since it is cyclic-free. For instance, removing generic arguments $\mathcal{A}_1, \mathcal{A}_2$ from our example will lead to a full tie between the alternatives because of lack of reasoning on the preferences.

2.1.4 Avoiding the Condorcet Paradox

This section is devoted to showing that the qualitative approach using abstract argumentation presented in this paper avoids the Condorcet paradox. The reason for which the Condorcet paradox is avoided lies on the reasoning capabilities of the ranking-completion argumentation framework. This type of framework along with Dung's preferred semantics help us construct the justified preference profile. This profile can be seen as a type of structured preferences in which a collective, according to Condorcet's method, cyclic preference will not exist when arguments posed during the deliberation phase are taken into account to compute the framework.

Theorem 1. *There are no voting cycles in any justified preference profile* $\succ_{\mathcal{JP}^p}$*.*

Proof. Please note that in the following, in order to avoid tedious notations, when we are referring to $\mathcal{JP}^p$ we are going to use the notation $\mathcal{JP}$.

Let $| \succ_{\mathcal{JP}} | = k$. In order to have a voting cycle, there must exist a justified preference profile $\succ_{\mathcal{JP}}$ where we have three alternatives $x, y, z \in A$ such that the following conditions hold:

$$|\mathcal{JP}_i \in \succ_{\mathcal{JP}}\colon\ x \succ_{\mathcal{JP}_i} y| > k/2, \tag{2.1}$$

$$|\mathcal{JP}_i \in \succ_{\mathcal{JP}}\colon\ y \succ_{\mathcal{JP}_i} z| > k/2, \tag{2.2}$$

$$|\mathcal{JP}_i \in \succ_{\mathcal{JP}}\colon\ z \succ_{\mathcal{JP}_i} x| > k/2 \tag{2.3}$$

Let $\mathcal{R}_{uv}$ be the subset of the set of rankings $\mathcal{R}$ in which u is ranked over v. Hence, all the possible sets of ranking arguments which are derived from $\mathcal{R}$ between these three alternatives are the following: $\mathbf{A}_{\mathcal{R}_{xy}}$, $\mathbf{A}_{\mathcal{R}_{yx}}$, $\mathbf{A}_{\mathcal{R}_{xz}}$,$\mathbf{A}_{\mathcal{R}_{zx}}$,$\mathbf{A}_{\mathcal{R}_{yz}}$, $\mathbf{A}_{\mathcal{R}_{zy}}$.

We are considering all the cases that may appear in the ranking-completion argumentation framework and as a consequence in the justified preference profile, with respect to these three alternatives and the preference relations arguments.

Case 1: When we have less than three preference relation arguments, we have two possible subcases. In the subcase where they can mutually attack each other, i.e., $\mathcal{A}_{xy}$ and $\mathcal{A}_{yx}$, it is easy to see that x is over y in half of the extensions rankings, and y is over x in the other half, which leads to a tie between x and y. In the other subcase it is easy to see that we cannot have all the ranking arguments $\mathcal{A}_{\mathcal{R}_{xyz}}$, $\mathcal{A}_{\mathcal{R}_{yzx}}$ and $\mathcal{A}_{\mathcal{R}_{zxy}}$ in the extensions out of only two preference relation arguments and hence, their corresponding rankings. Therefore, in both cases we do not have a cycle.

Case 2: When we have three preference relation arguments then in order to have a cycle we need to have these three arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$ and $\mathcal{A}_{zx}$ (or equivalently $\mathcal{A}_{yx}$, $\mathcal{A}_{zy}$ and $\mathcal{A}_{xz}$) to be included in the framework. This is valid because in order for alternative x to be ranked over y in the justified preference $\mathcal{JP}_i$, i.e., $x \succ_{\mathcal{JP}_i} y$ we need that the preference relation argument $\mathcal{A}_{xy}$ or a set of arguments of the form $\mathcal{A}_{x\mathcal{T}y}$ to be included in the corresponding extension and hence, in the argumentation framework. The set $\mathcal{A}_{x\mathcal{T}y}$ is the set of arguments that are transitively equivalent to $\mathcal{A}_{xy}$, i.e., $\mathcal{A}_{x\mathcal{T}y} = (\mathcal{A}_{x\omega} \cup \cdots \cup \mathcal{A}_{\psi y}), \omega, \psi \in A$. However, if we include arguments of the form $\mathcal{A}_{x\mathcal{T}y}$ then we have more than three arguments. Therefore, $\mathcal{A}_{xy}$ must be included in the framework and similarly, $\mathcal{A}_{yz}$ and $\mathcal{A}_{zx}$ must also be included.

We will show now that the cycle can be avoided when we have these three or more arguments. If we have only these three arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$ and $\mathcal{A}_{zx}$ included in the framework then all ranking arguments where $y \succ x$, $z \succ y$ and $x \succ z$ are attacked and thus, not included in the set of the extensions, i.e., the arguments of sets $\mathbf{A}_{\mathcal{R}_{yx}}$, $\mathbf{A}_{\mathcal{R}_{zy}}$ and $\mathbf{A}_{\mathcal{R}_{xz}}$. For the other possible ranking arguments, let us assume that the elements of $\mathbf{A}_{\mathcal{R}_{xy}}$ are included in the extensions. We can now distinguish between the following cases with regard to the rank of z compared to x and y.

- if $z \succ x \succ y$ then we have $z \succ y$ and hence, all the rankings in $\mathbf{A}_{\mathcal{R}_{zy}}$.
- if $x \succ z \succ y$ then we have $z \succ y$ and hence, all the rankings in $\mathbf{A}_{\mathcal{R}_{zy}}$.
- if $x \succ y \succ z$ then we have $x \succ z$ and hence, all the rankings in $\mathbf{A}_{\mathcal{R}_{xz}}$.

We can see that in all the cases where $x \succ y$ leads to arguments rankings that are not included in the extensions and hence, the elements of $\mathbf{A}_{\mathcal{R}_{xy}}$ are not included in the set of the extensions. Similarly, we can see that $\mathbf{A}_{\mathcal{R}_{zx}}$ and $\mathbf{A}_{\mathcal{R}_{yz}}$ are neither included in the set of extensions. Thus, the only ranking argument which is included in the extensions is the blank vote which leads in the avoidance of voting cycles.

The next step of the proof is to consider all the cases with respect to these three alternatives with the addition of the remaining preference relations arguments. We will also show that for the remaining cases the voting cycles are avoided.

Case 3: The following preference relation arguments are included in the framework: $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and $\mathcal{A}_{yx}$. The first extension we are computing contains arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and the blank vote $\mathcal{A}_B$. All the other extensions will contain the preference relation arguments $\mathcal{A}_{yx}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and the ranking arguments that are not attacked by them. Argument $\mathcal{A}_{yx}$ attacks the ranking arguments in $\mathbf{A}_{\mathcal{R}_{xy}}$, while $\mathcal{A}_{yz}$ and $\mathcal{A}_{zx}$ attack arguments in $\mathbf{A}_{\mathcal{R}_{zy}}$ and $\mathbf{A}_{\mathcal{R}_{xz}}$ respectively. Therefore, for arguments in $\mathbf{A}_{\mathcal{R}_{yx}}$ we distinguish as above between the following cases with regard to the rank of z compared to y and x. Similarly for the remaining non-attacked ranking arguments we distinguish with regard to the rank of x compared to y and z and the rank of y compared to z and x. Then, the only set of ranking arguments that is not attacked is the set $\mathbf{A}_{\mathcal{R}_{yzx}}$ which corresponds to the subset of rankings $\mathcal{R}_{yzx}$, i.e., all the rankings where $y \succ z \succ x$. Therefore the justified preference profile is composed of the blank vote and $\mathcal{R}_{yzx}$, which leads to no cycles.

A similar approach can be used when the following preference relation arguments are included in the framework: $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and $\mathcal{A}_{zy}$ (or $\mathcal{A}_{xz}$ resp.). It is easy to see that the only set of ranking arguments that is not attacked is the set $\mathbf{A}_{\mathcal{R}_{zxy}}$ ($\mathbf{A}_{\mathcal{R}_{xyz}}$ resp.) which corresponds to the subset of rankings $\mathcal{R}_{zxy}$ ($\mathcal{R}_{xyz}$ resp.), i.e., all the rankings where $z \succ x \succ y$ ($x \succ y \succ z$ resp.). Therefore the justified preference profile is composed of the blank vote and $\mathcal{R}_{zxy}$ ($\mathcal{R}_{xyz}$ resp.), which leads to no cycles.

Case 4: The following preference relation arguments are included in the framework: $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$, $\mathcal{A}_{yx}$ and $\mathcal{A}_{zy}$. The first extension we are computing contains arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and the blank vote $\mathcal{A}_B$. We have $k/3$ extensions that contain the preference relation arguments $\mathcal{A}_{yx}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and the ranking arguments that are not attacked by them. As in Case 3, the only set of ranking arguments that is not attacked is the set $\mathbf{A}_{\mathcal{R}_{yzx}}$ which corresponds to the subset of rankings $\mathcal{R}_{yzx}$, i.e., all the rankings where $y \succ z \succ x$. We also have $k/3$ extensions that contain the preference relation arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{zy}$, $\mathcal{A}_{zx}$ and the ranking arguments that are not attacked by them. Then, the only set of ranking arguments that is not attacked is the set $\mathbf{A}_{\mathcal{R}_{zxy}}$ which corresponds to the subset of rankings $\mathcal{R}_{zxy}$, i.e., all the rankings where $z \succ x \succ y$. Finally, we have $k/3$ extensions that contain the preference relation arguments $\mathcal{A}_{yx}$, $\mathcal{A}_{zy}$, $\mathcal{A}_{zx}$ and the ranking arguments that are not attacked by them. Then, the only set of ranking arguments that is not attacked is the set $\mathbf{A}_{\mathcal{R}_{zyx}}$ which corresponds to the subset of rankings $\mathcal{R}_{zyx}$, i.e., all the rankings where $z \succ y \succ x$. Therefore, in the justified preference profile we have the blank vote and $2 \cdot k/3$ justified preferences where $z \succ y$, $z \succ x$ and $y \succ x$, which leads to no cycles.

A similar approach can be used when the following preference relation arguments are included in the framework: $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$, $\mathcal{A}_{xz}$ and $\mathcal{A}_{yx}$ (or $\mathcal{A}_{zy}$ resp.). Here also, we have one extension with $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and $\mathcal{A}_{B}$; $k/3$ extensions that contain the preference relation arguments $\mathcal{A}_{yx}$ ($\mathcal{A}_{xy}$ resp.), $\mathcal{A}_{yz}$ ($\mathcal{A}_{zy}$ resp.), $\mathcal{A}_{zx}$ which correspond to all the rankings where $y \succ z \succ x$ ($z \succ x \succ y$ resp.); $k/3$ extensions that contain the preference relation arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{xz}$ which correspond to all the rankings where $x \succ y \succ z$; $k/3$ extensions that contain the preference relation arguments $\mathcal{A}_{yx}$ ($\mathcal{A}_{xy}$ resp.), $\mathcal{A}_{yz}$ ($\mathcal{A}_{zy}$ resp.), $\mathcal{A}_{xz}$ which correspond to all the rankings where $y \succ x \succ z$ ($x \succ z \succ y$ resp.). Hence, cycles are avoided since we have $2 \cdot k/3$ justified preferences where $y \succ z$, $x \succ z$ and $y \succ x$ ($z \succ y$, $x \succ z$ and $x \succ y$ resp.) and the blank vote.

Case 5: The last case is when all the preference relation arguments derived from x, y, z are included in the framework. There are two extensions containing the blank vote $\mathcal{A}_{B}$, i.e., the one that contains the arguments $\mathcal{A}_{xy}$, $\mathcal{A}_{yz}$, $\mathcal{A}_{zx}$ and the symmetric one that contains $\mathcal{A}_{yx}$, $\mathcal{A}_{zy}$, $\mathcal{A}_{xz}$. Regarding the rest of the extensions we have symmetrical cases, i.e., we have $k/6$ extensions containing the elements of $\mathbf{A}_{\mathcal{R}_{xyz}}$ and respectively $k/6$ extensions for each one of the possible ranking arguments. In this case we have a full equivalence, which means that there is a tie between x, y, z.

Recall that, due to Axiom 1, a generic argument cannot have an impact on preferences over different alternatives and hence, cannot attack two different preference relations arguments nor a ranking argument. Therefore, a generic argument can attack or defend a single preference relation argument. The latter is done by attacking the generic argument who attacks the preference relation argument. Since a generic arguments is only related to one preference relation argument, this leads to the conclusion that the outcome is not affected even if we have possible additions or removals of preference relation arguments in the extensions. An attack (or defence) of a generic argument can only affect one specific preference relation argument and removing (or adding) it does not change the analysis since we are coming back to the same cases described above when the consideration of only the preference relation arguments is taken into account.

Concluding, in all the above considered possible cases we have justified preference profiles without voting cycles and thus, we have completed the proof of Theorem 1.

□

2.1.5 Social Choice desirable properties

In the current section we explore the behaviour of the proposed approach towards desirable properties from the viewpoint of social choice. The Condorcet paradox is not the only deficiency that many well-known voting rules have. In order to evaluate further the method we are also referring to classical desirable properties from the social point of view which are often not satisfied.

Monotonicity

In the following section, we present our results about the property of monotonicity for our proposed method. A system is monotonic if a winning alternative remains the winning one in the new profile created after she is moved upward in the preferences of some of the agents.

Theorem 2. *The proposed method satisfies monotonicity when Condorcet-consistent voting rules are used for the aggregation of* $\succ_{\mathcal{JP}}$.

Proof. Suppose that w is the winning alternative in the original instance of the problem, i.e., having as input the preference profile $\succ_{PP}$. If we raise w in the preferences of some agents then we get as input the new profile $\succ'_{PP}$. In this profile compared to the original one $\succ_{PP}$ some preference relations arguments have been added and some have been removed because of moving w upwards.

For the analysis, we consider a reduced argumentation framework and the preference relation arguments without the justifications because it simplifies the proof and does not affect the result. Indeed, having two arguments $\mathcal{A}_{xy}$ and $\mathcal{A}'_{xy}$ in the same extension leads to the same ranking (linear order) of the justified preference $\mathcal{JP}_k$, where the linear order of $\mathcal{JP}_k$ belongs to the set $\mathcal{R}_{xy}$. Hence, let $AF(\succ_{PP})$ be the argumentation framework built from profile $\succ_{PP}$ but containing only one preference relation argument $\mathcal{A}_{xy}$ for declaring that an alternative x is preferred to y by an agent no matter what is the justification for this preference ($x \succ y$). We consider now two cases for the set $\mathbf{A}^*_{PR}$ of the preference relations arguments that have been added or removed in the $AF(\succ'_{PP})$ compared to $AF(\succ_{PP})$.

The **first case** is the one where the arguments in set $\mathbf{A}^*_{PR}$ are contained in both the original $AF(\succ_{PP})$ and the new framework $AF(\succ'_{PP})$. Observe that, this can happen because we consider the arguments without the justifications and if $\mathcal{A}_{wy}$ already exists adding $\mathcal{A}'_{wy}$ with a new justification will not create a new argument but will point to the existing one ($\mathcal{A}_{wy}$). In this case all the arguments point to existing ones and thus, the $AF(\succ'_{PP})$ equals to $AF(\succ_{PP})$. Therefore, we compute the same extensions from both frameworks and hence, the new justified preference profile, i.e., $\succ'_{\mathcal{JP}}$ is the same as the original one $\succ_{\mathcal{JP}}$. Consequently, the aggregation of both $\succ'_{\mathcal{JP}}$ and $\succ_{\mathcal{JP}}$ gives the same outcome under any voting rule, hence, w remains the winner.

In the **second case**, a subset of arguments in $\mathbf{A}^*_{PR}$ are not contained in both the frameworks. All the arguments of type $\mathcal{A}_{wy}$ should be included in both frameworks otherwise w cannot be the Condorcet winner. Indeed if the arguments $\mathcal{A}_{wy}$ are not included in $AF(\succ_{PP})$ or $AF(\succ'_{PP})$ then the ranking arguments $\mathbf{A}_{\mathcal{R}_{w\cdots y}}$ and thus, the rankings $\mathcal{R}_{wy}$ where w is over y cannot exist. Then, the only case we can have is to either add or remove the arguments $\mathcal{A}_{yw}$. If this subset of arguments is removed and not contained in the $AF(\succ'_{PP})$ then all rankings in $\succ'_{\mathcal{JP}}$ where y is over w are also removed and thus, w still beats y in a pairwise comparison and remains the Condorcet winner. Finally, note that we cannot add the arguments $\mathcal{A}_{yw}$ since $AF(\succ'_{PP})$ is a preference profile where the winner w is moved upwards in the preferences and not y.

Therefore taking into account all the cases regarding the differences between profiles $AF(\succ_{PP}$ $)$ and $AF(\succ'_{PP})$, alternative w remains the winner. □

Homogeneity

In the following section, we present our results about the property of homogeneity for the proposed method. A system is homogeneous if the replication of the preference profile does not change the winning ranking of the alternatives.

Theorem 3. *The proposed method satisfies homogeneity for any voting rule used for the aggregation of $\succ_{\mathcal{JP}}$.*

Proof. Suppose that $\mathcal{R}$ is the winning ranking in the original instance of the problem, i.e., having as input the preference profile $\succ_{PP}$. If we replicate the original profile by n times then we get as input the new profile $\succ^n_{PP}$. It is easy to see that this profile produces the exact same set of preference relations arguments when compared to the original one. Hence, the argumentation framework built from it, i.e. $AF(\succ_{PP^n})$, is the same as the original one $AF(\succ_{PP})$ and thus, the new justified preference profile, $\succ^n_{\mathcal{JP}}$, is the same as the original one, $\succ_{\mathcal{JP}}$. Consequently, the aggregation of both $\succ^n_{\mathcal{JP}}$ and $\succ_{\mathcal{JP}}$ gives the same outcome under any voting rule and thus, the winning ranking $\mathcal{R}$ remains the same under both profiles. □

2.1.6 Conclusion and future work

In this paper, we have proposed a framework for decision-making that relies on the qualitative preferences which comes in contrast with the social choice methods which rely only on the quantitative aggregation of the individual preferences. The intuition behind our approach was to find a method that simulates a pre-voting, i.e., deliberation, phase in a collective decision and that takes into account the agents' expressed arguments. Hence, the proposed method produces arguments based on the preferences of the agents and the justifications behind these preferences. The aggregation of all the agents' arguments is computed based on the attacks between them and leads to possible extension(s) where each one of them depicts collectively justified preferences. When there is no consensus among agents' preferences, multiple extensions are computed and hence, a Condorcet method can be used to aggregate the collective justified preference profile. Due to the construction of the argumentation framework, the justified preference profile is a type of structured profile that can avoid the Condorcet paradox.

In terms of future work, we want to further extend our research towards the collective multi-criteria decision-making problem and get deeper in the integration of argumentation with computational social choice. We plan to explore techniques from computational social choice that will permit us to propose decision aiding procedures that can support group preferences and we believe that argumentation framework can provide us the reasoning behind the decision makers preferences. The combination of these two subfields of computer science will allow us to explain the decisions rationally and thus, we want to propose more procedures for collective decisions that will have more chances to be accepted by the society. To strengthen this view we plan to propose quantitative methods that can compare and evaluate the different decision-making procedures. Also, it is left for future work to study the complexity of our approach; in general abstract argumentation is subject to quite bad complexities (see for instance [48] and [49]), but solvers such as CoQuiAAS[3] might allow to handle quite large argumentation systems as shown in [86]. Therefore, it would be interesting to adapt this kind of solvers to the ranking-completion argumentation framework in order to assess the practical relevance of our approach.

2.2 A Voting Argumentation Framework: Considering the Reasoning behind Preferences

Abstract

One of the most prominent ways to reach an acceptable collective decision in normal group settings is the employment of routines and methods of social choice theory. The classical social choice setting is the following: each agent involved in the decision expresses her preferences about a given set of alternatives in the form of a linear order on them. Then, the group's aggregated decision is the outcome of the application of a voting rule to the input's preferences. However, there are instances where social choice on its own cannot provide proper solutions. For example, there are decision problems where the outcome has to be based on the reasoning behind agents' preferences, rather than the unjustified preferences itself. Hence, our research motivation is the practical case where agents' rationale is needed for the decision

[3] `http://www.cril.univ-artois.fr/coquiaas/`

outcome. In this paper, we explore how the agents' rationale can be formulated inside the classical voting setting. Therefore, we propose a decision-making procedure based on argumentation and preference aggregation which permits us to explore the effect of reasoning and deliberation along with voting for the decision process. We quantify the deliberation phase by defining a new voting argumentation framework, that uses vote and generic arguments, and its acceptability semantics based on the notion of pairwise comparisons between alternatives. We prove for these semantics some theoretical results regarding well-known properties from argumentation and social choice theory.

2.2.1 Introduction

Collective decision-making in the context of multi-agents systems is a well-studied problem where many possible research approaches have been proposed in the literature for solving it. An overview of the research approaches can be found in this survey [34]. The procedure followed to reach a group decision is a complex task, in which there are many parameters that affect decision makers judgement. The key to making a "good" collective decision is knowing and hence, the agents (decision-makers) should have full knowledge on the different parameters that entail collective decisions.

In the decision-making literature it is widely believed that in order to confirm that the chosen decision outcome is the best one, the decision makers should believe that this is the best outcome, and have reasons to believe this. Using social choice theory we secure the first condition: agents express their individual preferences on the decision outcomes and voting methods provide the means in order for agents to believe that their aggregated preference (outcome) is the best, i.e., fairest according to their preferences. Our motivation comes from fulfilling the second condition and in order to do that we have to take into account the reasoning behind the preferences and thus, deliberation and argumentation play an important role. Hence, the scope of this paper is to fulfill the central decision-making problem, which is to help decision makers produce "better" collective decision outcomes. "Better" decisions is a very broad term and the goal of many research papers on this domain. In this paper, we will study on how collective decision-making is helped with the intersection of argumentation along with deliberation in social choice theory. Thus, we will focus on a social choice theoretic approach for multi-agent decision making enriched with an argumentation framework.

Social choice theory can be applied to multi-agent systems [50] where voting can provide the classical means for aggregating the individual agents' preferences into a collective decision. In the original setting we have a set of *agents* and a set of *alternatives*. Each agent expresses her preference as a total order over a set of alternatives, and then the group's preference is computed from the individual preferences using a *voting rule*. A more analytic description of the voting problem and the social choice fundamentals can be found in the Handbook of Computational Social Choice [33].

As previously mentioned, we believe that collective decision making should also rely on the reasoning the agents provide when expressing their preferences. In order to strengthen this view we can mention here that it is common in many occasions that agents lie in expressing preferences in favor of specific alternatives when it is not possible to justify their reasons. The same has also been noticed even if agents provide reasoning but there is no deliberation phase. One such example are the reviews and ratings agents provide in sites like Google, Amazon, etc., where in some cases the percentage of fake/questionable reviews for a category of alternatives/products can reach 67% [4]. Hence, it is logical to assume that the agents should not only give their preferences but provide also a reasoning about their preferences so that one can debate. Therefore, it is reasonable to search for a way to interpret the preferences and the reasoning behind them and an argumentation framework [22, 47] seems to be a rational approach to do that. Hence, in order to fulfill a collective decision mechanism that considers both reasoning and deliberation we propose a decision-aiding procedure which combines argumentation with computational social choice.

Argumentation theory is widely used in the multi-agent decision-making context, e.g., [7, 57, 53] due to its ability for reasoning with incomplete and conflicting information (such as differences in opinions). An argumentation framework is based on the construction, the exchange and the evaluation of interacting arguments, where various semantics are defined in the literature to assess the acceptability of sets of arguments. Hence, an argumentation framework where agents provide arguments with their preferences can "correct" the "false" or "fake" information that can appear. The way to measure the "false" information included in agents' preferences is by introducing the notion of *attacking power* of arguments. It is a function that quantifies the attacking strength of argument(s) exposed during the deliberation phase towards an argument stating an agent's preference. For example, if a preference argument of an agent is attacked by many arguments which are revealed during deliberation that is most likely to mean that this preference is not truthful and thus its power for the collective decision should be reduced. Therefore, we introduce a method which takes into account the attacking power of the deliberation phase to reach a collective decision.

Concluding, it is our belief that enhancing the collective decision-making procedure with a voting argumentation framework can benefit the procedure in the following ways. First, agents' justifications for preferences, which are depicted in the construction of the argumentation framework, can provide the reasoning which can serve as the rational explanation of the collective decision. Second, an argumentation framework can model the deliberation phase prior to the application of voting for making a group decision. This modelling permits us to construct a preference profile that is "justified", since it refers to the agents' preferences and their justifications. The justified preference profile is a type of structured profile which is the outcome of

[4] https://www.washingtonpost.com/business/economy/how-merchants-secretly-use-facebook-to-flood-amazon-with-fake-reviews/2018/04/23/5dad1e30-4392-11e8-8569-26fda6b404c7_story.html

a pre-voting debate phase that consists of a deliberation procedure where agents reveal their preferences and justifications. The objective is to fairly aggregate the justified viewpoints of the agents and hence, the justified preference profile can be reported to a voting rule for computing the decision outcome.

Our work. Seminal to our research is the work of [25] which first presented an argumentation framework based on agents' preferences for the voting problem from a *qualitative* perspective. Based on the notions of this paper we present a novel *quantitative* procedure by designing a special kind of argumentation framework, the Voting Argumentation Framework ($\mathcal{VAF}$) and its corresponding semantics, which are called pairwise comparison semantics. The proposed semantics take into account the deliberation[5] phase in terms of quantifying the attacking power of arguments on the justification of the arguments produced by agents' preferences. We then compute the acceptability of the vote arguments which define the new profile, called the justified preference profile, that takes into account the justification and deliberation phase. The justified preference profile is the outcome of a quantitative argumentation framework and its semantics and contains now all the "corrected" preferences of the agents. Thus, a voting rule can be applied to aggregate these preferences, which gives us the motivation to study social choice theoretic properties for the justified preference profile and prove under which conditions they can be satisfied. Finally, we look into properties that $\mathcal{VAF}$ and its corresponding semantics should satisfy from an argumentative perspective.

Related work. The intersection of argumentation and deliberation in social choice theory for "better" decision outcomes is a recently developed field. As already noted "better" can be defined in many ways. For example, in the work of [70] "better" refers to outcomes where majority cycling can be bypassed through single-peakedness. They show that deliberation can protect against majority cycles by presenting an empirical test using real data and use-cases from Deliberative Polls. The results show that the preferences after deliberation phase are closer, in terms of a particular proximity function, to being single-peaked. On the same path, i.e., decision outcomes based on deliberation that avoid majority cycling and related paradoxes, [25] design a qualitative model based on an argumentation framework that is built from the justifications of the agents pairwise preferences. They use Dung's "preferred" semantics to compute extensions which provide the justified preference profile. The justified preference profile is computed through a qualitative method where the strength of votes, i.e, the number of times that a pairwise comparison appears in the original preference profile, is not taken into account for the aggregation. Hence, this method focuses on cases where the decision outcome is independent

[5]It should be noted that, similarly to [25], our work does not address deliberation protocols such as [65] directly. Indeed, we assume that the argumentation framework is obtained as a result of a unspecified deliberation protocol. This allows to abstract the approach from the representation of the deliberation itself.

of the aggregation of agents' preference relations. In our approach we instead focus on cases where the strength of the votes are included in the collective decision. They prove that the justified preference profile permits a type of structured preferences where the Condorcet paradox is avoided. Another work for exploring the effect of deliberation and its benefits in voting is the paper of [52]. The authors in this paper present a sequential deliberation protocol, seen from a game-theoretic perspective, where agents negotiate in pairs and collaboratively propose outcomes that appeal to both of them. They describe a method where the space of preferences is defined by a median graph and prove that sequential deliberation is 1.208-approximate to the optimal social cost on such graphs. They also study the Pareto-efficiency property from a social choice perspective and prove that the outcome of sequential deliberation is ex-post Pareto-efficient on a median graph, i.e., that there is no other alternative that has at most that social cost for all agents and strictly better cost for one agent.

There is also some significant research on the intersection of social choice and argumentation which is proximately related to this work. Most of the works towards this research study the problem from an argumentative perspective and deal with collective argumentation. The studied problem refers to aggregating individual argumentation frameworks to a collective one. The aggregation mechanisms provided to compute the collective frameworks rely on social choice, which provides the means to accomplish that. A nice example towards this research direction is the work of [42] for merging argumentation systems. They focus on scenarios in which some agents are able to consider arguments not known by other agents and disagree on the attack relation. A three-step process is proposed where in the first one, each attack relation is consensually expanded to become a partial system over the set of arguments. In the second step the merging is done by generating a class of argumentation systems that are at the shortest "distance" of the ones in the profile. In the final step the acceptable arguments are selected. Another example towards this direction is the research of [44] where specific merging operators based on extensions, though in combination with a framework-wise merging process. The authors study the generation of the argumentation framework resulting from a merging process. There has been also notable research on the combination of Judgement Aggregation, which is one of the main problems in social choice, and argumentation where Pigozzi et al. have extensively studied the problem. In [75] the author proposes an aggregation procedure, called argument-based, for the case in which the outcome is a set of arguments, combining features of premise and conclusion-based procedures. [76] study the same problem looking for axioms that characterize the aggregation procedure and conditions such that the Condorcet Paradox is avoided. They show that a condition called premise independence of irrelevant propositional alternatives guarantees the existence of consistent ways of aggregating judgments. [36] employ an argumentation approach to judgment aggregation which satisfies standard judgment aggregation postulates and also avoids the problem of individual agents having to become committed to a group judgment that is in conflict with their own preferences. Another prominent work combining argumentation and Voting is the one of [68] where they propose Social abstract argumentation framework, which is based on Dung's Abstract argumentation framework but also incorporates social voting. They propose a class of semantics for the Social abstract argumentation framework and prove some important properties regarding Social Networks. An explorative survey for collective argumentation is provided by [27].

On a related research path, recently, there has been some work on the reasoning behind preferences but not from an argumentative and social choice perspective. For example, [46] propose a reason-based theory of rational choice where agents' preferences are determined by their motivating reasons and clarify the relationship between deliberation for reasons and for rational choices. Another work on the same path is the one of [74] where they develop a modal logic for reasoning about preferences that depends on a set of motivational properties and also show that reasoning systems and algorithms developed for modal logic can be employed for reasoning about reason-based preferences.

2.2.2 Preliminaries

In the following section, we are defining several notions and notations that will be used later in the paper.

Social Choice Theory

We consider a set of $N = \{1, \dots, n\}$ *agents* and a set of *alternatives* A, $|A| = m$. Each agent $i \in N$ has preference relations ($\succ$) over the alternatives denoted with $x \succ_i y$ which means that agent i *prefers* alternative x to y. We define that each irreflexive preference relation satisfies transitivity, antisymmetry and comparability and hence, the set of all the preference relations for agent i produces a linear (strict total) order $\succ_i$ on A, i.e., the ranking of agent i over the alternatives. Let $\mathcal{L}_A$ be the set of linear orders over A. A *preference profile* $\succ_{PP} = \langle \succ_1, \dots, \succ_n \rangle \in \mathcal{L}_A^n$ is a collection of the linear orders for all the agents. A *voting rule* is a mapping $f : \mathcal{L}_A^n \to 2^A \setminus \{\emptyset\}$ from preference profiles to nonempty subsets of alternatives, which designates the winner(s) of the election. For two alternatives $x, y \in A$, and $\succ_{PP} \in \mathcal{L}_A^n$, alternative x *beats* y in a *pairwise comparison* if $|\{i \in N : x \succ_i y\}| > n/2$, that is, if a (strict) majority of agents prefer x to y. The winner according to the *Condorcet method* [40], i.e. the *Condorcet winner*, is an alternative that beats every other alternative in a pairwise comparison. The *Condorcet paradox* as defined by [26] (also known as voting paradox or the paradox of voting) is a situation in which the application of the Condorcet method to a preference profile can lead to a voting cycle, and hence a Condorcet winner can not be declared. A *voting cycle* occurs when we have 3 alternatives x, y, z such that $|\{i \in N : x \succ_i y\}| > n/2$, $|\{i \in N : y \succ_i z\}| > n/2$, and

$|\{i \in N : \ z \succ_i x\}| > n/2$.[6]

Argumentation

In order to be general with regards to the deliberation step, we build upon the abstract argumentation framework proposed by [47]:

Definition 5 (Argumentation framework [47])**.** *An argumentation framework (AF) is a pair* $(\mathbf{A}, \mathbf{R})$*, where* $\mathbf{A}$ *is a finite nonempty set of arguments and* $\mathbf{R}$ *is a binary relation on* $\mathbf{A}$*, called attack relation. Let* $\mathcal{A}, \mathcal{B} \in \mathbf{A}$*,* $\mathcal{A}\mathbf{R}\mathcal{B}$ *means that* $\mathcal{A}$ *attacks* $\mathcal{B}$*.*

Definition 6 (Ranking-based semantics [29])**.** *Given an AF* $(\mathbf{A}, \mathbf{R})$*, a ranking-based semantics* σ *associates a ranking* $\succeq^\sigma$ *on* $\mathbf{A}$*.* $\succeq^\sigma$ *is a preorder (a reflexive and transitive relation) on* $\mathbf{A}$*. For* $a, b \in \mathbf{A}$ $a \succeq^\sigma b$ *means that* a *is at least as acceptable as* b*.*

Definition 7 (Path, attackers and defenders [29])**.** *Given an AF* $(\mathbf{A}, \mathbf{R})$*, and* $\mathcal{A}, \mathcal{B} \in \mathbf{A}$*. A path* $p_{\mathcal{B},\mathcal{A}}$ *is a sequence* $s = \langle a_0, \cdots, a_n \rangle$ *of arguments where* $a_0 = \mathcal{A}$ *,* $a_n = \mathcal{B}$ *and* $\forall i < n$ *we have that* $(a_{i+1}, a_i) \in \mathbf{R}$*. The length of the path is denoted by* ℓ_p*. Note that* $\ell_p = n$*. A defender of an argument* $\mathcal{A}$ *is an argument situated at the beginning of an even-length path leading to* $\mathcal{A}$*. Respectively, an attacker of* $\mathcal{A}$ *is an argument situated at the beginning of a path of odd length. We denote the multiset of defenders and attackers of* $\mathcal{A}$ *by* $Def_n(\mathcal{A}) = \{b \mid \exists p_{\mathcal{B},\mathcal{A}},\ \ell_p \in 2\mathcal{N}\}$ *and* $Att_n(\mathcal{A}) = \{b \mid \exists p_{\mathcal{B},\mathcal{A}},\ \ell_p \in 2\mathcal{N} + 1\}$ *respectively. The direct attackers of* $\mathcal{A}$ *are the arguments in* $Att_1(\mathcal{A})$*. An argument* $\mathcal{A}$ *is defended if* $Def_2(\mathcal{A}) \neq \emptyset$*.*

Based on these notions, we can now present the Voting Argumentation Framework combining the strengths of social choice and argumentation.

2.2.3 The Voting Argumentation Framework

Construction of the Voting Argumentation Framework

In order to take advantage of the reasoning capabilities of Argumentation Framework in social choice we define our model by constructing a special argumentation framework adapted for social choice and voting. In the following we describe the construction of this specialized framework, which we will call *Voting Argumentation Framework*, i.e, $\mathcal{VAF}$.

We are going to distinguish between two types of arguments: *"vote"* arguments and *"generic"* arguments. We describe their role and the attacks that can occur between them in the following paragraphs.

[6]We are assuming an odd number of agents.

Vote arguments A vote argument $\mathcal{A}_{r_{i,j}}$ represents the argument which considers the total order, i.e., ranking r_i produced by agent j and the justification provided by this agent for each of the pairwise comparisons included in her total order. We denote by $\mathbf{A}_{\mathcal{R}} = \{\mathcal{A}_{r_{i,j}}, \forall j \in N\}$ the set of all the vote arguments and by $\mathbf{A}_{\mathcal{R}_{xy}}$ the set of vote arguments where the preference relation $x \succ y$ occurs in the ranking r_i.

Vote arguments cannot be attacked by other vote arguments, since it is natural to assume that for each distinct agent j a different vote argument is produced representing her preferences. Hence, two different votes given by two different agents are not considered inconsistent.

Example 1. *Assume a decision problem with three agents $\{v_1, v_2, v_3\}$ and three alternatives $\{c_1, c_2, c_3\}$. The agents after a deliberation phase provide the following preferences along with a justification:*

- v_1: $c_1 \succ c_2 \succ c_3$
- v_2: $c_2 \succ c_3 \succ c_1$
- v_3: $c_3 \succ c_1 \succ c_2$

These preferences and their justifications are respectively represented by vote arguments $\mathbf{A}_{\mathcal{R}} = \{\mathcal{A}_{r_{1,v_1}}, \mathcal{A}_{r_{2,v_2}}, \mathcal{A}_{r_{3,v_3}}\}$.

Generic arguments Generic arguments represent the deliberation phase and regroup all the other possible arguments that can arise during a debate. In particular, those arguments are able to attack other generic arguments and vote arguments. Indeed, giving a reason contradicting a preference $x \succ y$ of agent j triggers an attack on the vote argument $\mathcal{A}_{r_{i,j}}$. We denote this generic argument as a direct attacker of $\mathcal{A}_{r_{i,j}}$, i.e., $da_{r_{i,j}} \in Att_1(\mathcal{A}_{r_{i,j}})$. The generic argument, denoted by $g^k_{r_{i,j}}$, which is situated in the beginning of a path leading to $\mathcal{A}_{r_{i,j}}$ can either attack or defend a vote argument $\mathcal{A}_{r_{i,j}}$. Note that even if the premise of an argument $g^k_{r_{i,j}}$ is the same as argument $g^k_{r_{i',j'}}$, we consider them as different arguments as they correspond to the vote of a different agent, i.e., j versus j'. Hence, a generic argument $g^k_{r_{i,j}}$ can not have a path leading to two different vote arguments. By k we denote the index of generic arguments attacking/defending $\mathcal{A}_{r_{i,j}}$. We denote by G the set of all the generic arguments and by $G_{r_{i,j}}$ the set of generic arguments attacking $\mathcal{A}_{r_{i,j}}$.

Example 1 (cont.)**.** *Assume now that the agents enunciated eight generic arguments $G = \{g^1_{r_{i,v_i}}, \ldots, g^8_{r_{i,v_i}}\}$, $i \in [1, 3]$. Figure 2.3 presents the argumentation framework $(\mathbf{A}_{\mathcal{R}} \cup G, \mathbf{R})$, where $\mathbf{R}$ is represented by the arrows between the arguments.*

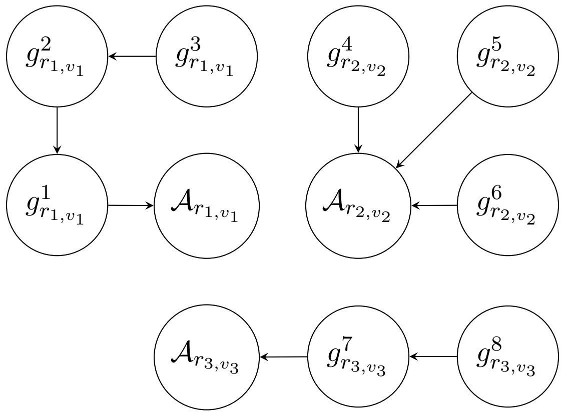


Figure 2.3: Voting argumentation framework of Example 1.

A quantitative model for decision making based on the Voting Argumentation Framework

In the following we use $\mathcal{VAF}$ in order to present a quantitative model for social choice and decision making problems that takes into account the reasoning behind the preferences. We start by defining our proposition for a new kind of semantics for computing the acceptability of the vote arguments. We call them Pairwise Comparison Semantics and can be seen as a kind of ranking-based semantics specially adapted to fit in the voting setting. The intuition is the same though, as each vote argument has a degree which denotes its strength, and hence the level of acceptability in the outcome. We are then computing a new preference profile taking into account the strength and the acceptability of the vote arguments. This new profile will be thus "justified" as it is based on the outcome of the voting argumentation framework.

Pairwise Comparison Semantics Let $P_{r_{i,j}}$ the set of paths $p_{g,\mathcal{A}_{r_{i,j}}}$ of attacks starting from a generic argument $g \in G$ and leading to vote argument $\mathcal{A}_{r_{i,j}}$. The attacking power of a generic argument $g^k_{r_{i,j}}$, which is the starting point of a path $p_{g^k_{r_{i,j}},\mathcal{A}_{r_{i,j}}}$, on a vote argument $\mathcal{A}_{r_{i,j}}$ is denoted by $ap(g^k_{r_{i,j}})$ and is computed by the following formula.

$$ap(g^k_{r_{i,j}}) = \begin{cases} \frac{1}{\frac{m\cdot(m-1)}{2}} & \text{if an odd-length path } p_{g^k_{r_{i,j}},\mathcal{A}_{r_{i,j}}} \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

Example 1 (cont.)**.** *The attacking powers of the generic arguments that are the starting nodes of the paths attacking the vote arguments are the following.*

$$ap(g^3_{r1,v_1}) = \frac{1}{3}$$

$$ap(g^4_{r_{2,v_2}}) = ap(g^5_{r_{2,v_2}}) = ap(g^6_{r_{2,v_2}}) = \frac{1}{3}$$
$$ap(g^8_{r_{3,v_3}}) = 0$$

Note that when the attacking power of a generic argument is 0 then this argument is defending the vote argument. The intuition for computing the attacking power function by the above formula is that in each vote argument there are $\frac{m \cdot (m-1)}{2}$ pairwise comparisons between the alternatives, where m is the number of the alternatives. We assume that each generic argument refers to the justification of one pairwise comparison each time. Hence, the weight of each vote argument is reduced by $\frac{1}{\frac{m \cdot (m-1)}{2}}$ when it is attacked by a generic argument. What we are actually interested in is the effect a generic argument has on the weight of the vote argument. Hence, the reason for denoting positive attacking power on a generic argument only if there exists a path of odd length starting from it, is that having such a path affects the weight of the vote argument as there exists an active direct attacker on the vote argument. In the case where there exists an even path means that the direct attacker is not active so the attacking power of the path is 0. Note that according to Dung's preferred semantics an even path would defend the direct attacker of a vote argument even if an odd path exists. In our case though, the semantics we use are not binary as Dung's, i.e., an argument can either be included or excluded from an extension, but instead we propose semantics where the level of acceptability of an argument depends on its weight, which is reminiscent of ranking semantics [29]. Hence, having an odd path is sufficient for the vote argument to decrease its acceptability value. It is reasonable to assume here that if one wants to define the effect of the deliberation phase, which is reflected through the generic arguments, in a different way then the attacking power function should be changed.

The attacking power of the set of generic arguments attacking $\mathcal{A}_{r_{i,j}}$, denoted by $ap(G_{r_{i,j}})$, is the sum of all the generic arguments attacking it, hence $ap(G_{r_{i,j}}) = \sum_k ap(g^k_{r_{i,j}})$.

Example 1 (cont.)**.** *The attacking power of the set of generic arguments attacking the vote arguments is the following for each one of them.*

$$ap(G_{r_{1,v_1}}) = \frac{1}{3}$$
$$ap(G_{r_{2,v_2}}) = 1$$
$$ap(G_{r_{3,v_3}}) = 0$$

Since our goal is to design an Argumentation Framework towards social choice and voting we care about joining together the vote arguments $\mathbf{A}_{r_i}$ that correspond to the same total order r_i rather than the single vote argument $\mathcal{A}_{r_{i,j}}$ produced by agent j itself. We call *unification (coalition)* of arguments this joining of arguments $\mathcal{A}_{r_{i,j}}$, who have the same ranking r_i, into a meta-argument $\mathbf{A}_{r_i}$ with higher weight. We define as $ap(G_{r_i}) = \sum_j ap(G_{r_{i,j}})$ the attacking power of the generic arguments attacking $\mathbf{A}_{r_i}$. The attacking power of the whole deliberation phase is defined as $ap(G) = \sum_{r_i} ap(G_{r_i})$.

The weight of each vote argument is initially 1, hence $w(\mathcal{A}_{r_{i,j}}) = 1$. A total order r_i can appear $|r_i|_\succ$ times in the preference profile $\succ$, which means that $|r_i|_\succ$ agents have a preference order $|r_i|$. It is easy to see that if we sum up the initial weights of all the vote arguments expressing votes with total order r_i, then we get the number of appearances of $|r_i|_\succ$ in the preference profile. Hence, $|r_i|_\succ = \sum_j w(\mathcal{A}_{r_{i,j}})$, where j denotes an agent voting for r_i.

For each vote argument $\mathcal{A}_{r_{i,j}}$ we define its degree for the acceptability semantics as

$$d(\mathcal{A}_{r_{i,j}}) \quad = \quad \max\left\{0, w(\mathcal{A}_{r_{i,j}}) - ap(G_{r_{i,j}})\right\}.$$

Example 1 (cont.)**.** *The weight of each vote argument is initially 1, hence* $w(\mathcal{A}_{r_{1,v_1}}) = w(\mathcal{A}_{r_{2,v_2}}) = w(\mathcal{A}_{r_{3,v_3}}) = 1$. *For each vote argument we compute its degree:*

$$\begin{aligned} d(\mathcal{A}_{r_{1,v_1}}) &= 1 - \frac{1}{3} = \frac{2}{3} \\ d(\mathcal{A}_{r_{2,v_2}}) &= 1 - 1 = 0 \\ d(\mathcal{A}_{r_{3,v_3}}) &= 1 - 0 = 1 \end{aligned}$$

Computing the justified preference profile under $\mathcal{VAF}$ and Pairwise Comparison Semantics It is possible to compute the set of "coherent preferences", i.e., the justified preference profile, by using the defined semantics on the voting argumentation framework. The above mentioned semantics define the acceptability degree of each vote argument in the justified preference profile $\succ_{\mathcal{JP}}$ under $\mathcal{VAF}$. For simplicity and onwards, when we refer to $\succ_{\mathcal{JP}}$ computed by $\mathcal{VAF}$ and the pairwise comparison semantics, we will use just the $\succ_{\mathcal{JP}}$ symbol. In order to build the $\succ_{\mathcal{JP}}$ we take into account the degree of each vote argument. For each total order r_i we compute the utility/acceptability $uv(r_i)$ value to denote its strength in the $\succ_{\mathcal{JP}}$. For the computation we take into account the acceptability degrees of each of the vote arguments $\mathcal{A}_{r_{i,j}}$ that refer to this total order (r_i), i.e., the degree of meta-argument $\mathbf{A}_{r_i}$ after the unification. Hence $uv(r_i) = d(\mathbf{A}_{r_i}) = \sum_{\forall j} d(\mathcal{A}_{r_{i,j}})$. It is easy to verify that $uv(r_i) = |r_i|_\succ - ap(G_{r_i})$. The number of times a ranking r_i appears in the justified preference profile $\succ_{\mathcal{JP}}$ is the ratio of the utility value of r_i over the sum of all the utility values multiplied by the total number of agents n' in $\succ_{\mathcal{JP}}$. Hence, $|r_i|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}$.

The total number of agents in the $\succ_{\mathcal{JP}}$ is computed as following. Note that N' is the set of agents in the $\succ_{\mathcal{JP}}$.

$$n' = \begin{cases} \sum_{\forall r_i} uv(r_i) & \text{if } \forall uv(r_i) \in \mathcal{N} \\ \dfrac{\sum_{\forall r_i} uv(r_i)}{gcd(uv(r_i), \forall r_i \in N')} & \text{if } \forall uv(r_i) \in \mathcal{Q} \setminus \mathcal{N} \end{cases}$$

When it is clear in the context and for notation simplicity, we refer to *gcd* for denoting the $gcd(uv(r_i), \forall r_i \in N')$, which is the greatest common divisor of all the utility values $uv(r_i)$ that belong to the set of agents N' in the $\succ_{\mathcal{JP}}$.

The need to have a integer number of agents leads to considering the multiplication of $\sum_{\forall r_i} uv(r_i)$ by the *gcd*.

Example 1 (cont.). *In the following, we compute the justified preference profile $\succ_{\mathcal{JP}}$ by defining the number of times a ranking r_i appears in it. For each total order r_i we compute the utility $uv(r_i)$ taking into account its acceptability degree. Hence, we have that $uv(r_1) = \frac{2}{3}, uv(r_2) = 0, uv(r_3) = 1$. Therefore, $\sum_{\forall r_i} uv(r_i) = \frac{5}{3}$ and $gcd(1, 0, \frac{2}{3}) = \frac{1}{3}$. The total number of agents n' in $\succ_{\mathcal{JP}}$ is $n' = \frac{5/3}{1/3} = 5$. Therefore,*

$$\begin{aligned} |r_1|_{\succ_{\mathcal{JP}}} &= 5 \cdot \frac{2/3}{5/3} &= 2 \\ |r_2|_{\succ_{\mathcal{JP}}} &= 5 \cdot \frac{0}{5/3} &= 0 \\ |r_3|_{\succ_{\mathcal{JP}}} &= 5 \cdot \frac{1}{5/3} &= 3 \end{aligned}$$

That means we have 2 agents with ranking $c_1 \succ c_2 \succ c_3$ and 3 agents with $c_3 \succ c_1 \succ c_2$ in the $\succ_{\mathcal{JP}}$.

2.2.4 Properties of $\mathcal{VAF}$ under Pairwise Comparison Semantics

In this section, we are going to study some desirable properties that should be satisfied by any voting argumentation framework.

$\mathcal{VAF}$ Desirable Properties from an argumentative perspective

We will study the properties presented in the literature of argumentation and ranking-based semantics that make sense to be satisfied by a Voting Argumentation Framework and its corresponding semantics. An overview of the desirable properties for ranking-based semantics can be found in [29]. We believe that these properties should be taken into account when one wants to compute the social choice outcome. Under this perspective, we slightly change the definitions of the properties of Cardinality and Defense Precedence. Hence, we call the modified properties as Weak Cardinality Precedence and Weak Defense Precedence.

Definition 8 (Weak Cardinality Precedence)**.** *The greater the number of direct attackers for a vote argument, the weaker the level of acceptability of this argument. Given two vote arguments* $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$ *s.t.* $d(\mathcal{A}) \neq 0$ *and* $d(\mathcal{B}) \neq 0$*,*

$$|Att_1(\mathcal{A})| < |Att_1(\mathcal{B})| \Rightarrow \mathcal{A} \succ \mathcal{B}.$$

Theorem 1. *The justified preference profile computed by* $\mathcal{VAF}$ *under pairwise comparison semantics* ($\succ_{\mathcal{JP}}$) *satisfies Weak Cardinality Precedence.*

Proof. The degree $d(\mathcal{A})$ of an argument $\mathcal{A}$ defines the level of its acceptability in the framework. Thus, it suffices to show that given two vote arguments $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$ s.t. $d(\mathcal{A}) \neq 0$ and $d(\mathcal{B}) \neq 0$, if $|Att_1(\mathcal{A})| < |Att_1(\mathcal{B})|$ then $d(\mathcal{A}) > d(\mathcal{B})$.

If $|Att_1(\mathcal{A})| < |Att_1(\mathcal{B})|$ then $ap(G_{\mathcal{A}}) < ap(G_{\mathcal{B}})$. The degree of a vote argument $\mathcal{Y}$ is given by

$$d(\mathcal{Y}) \quad = \quad \max\{0, w(y) - ap(G_{\mathcal{Y}})\}.$$

Since $d(\mathcal{A}) \neq 0$ and $d(\mathcal{B}) \neq 0$, we have that $d(\mathcal{A}) = w(\mathcal{A}) - ap(G_{\mathcal{A}})$ and $d(\mathcal{B}) = w(\mathcal{B}) - ap(G_{\mathcal{B}})$. Also, we have that $w(\mathcal{A}) = w(\mathcal{B}) = 1$. Summing up, we have that $d(\mathcal{A}) > d(\mathcal{B})$. □

Definition 9 (Weak Defense Precedence)**.** *For two vote arguments with the same number of direct attackers, a defended argument is ranked higher than a non-defended argument. Given two vote arguments* $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$ *s.t.* $d(\mathcal{A}) \neq 0$ *and* $d(\mathcal{B}) \neq 0$*,*

$$\begin{array}{l} |Att_1(\mathcal{A})| = |Att_1(\mathcal{B})|, \\ Def_2(\mathcal{A}) \neq \emptyset \text{ and } Def_2(\mathcal{B}) = \emptyset \end{array} \quad \Rightarrow \mathcal{A} \succ \mathcal{B}.$$

Theorem 2. *The justified preference profile computed by* $\mathcal{VAF}$ *under pairwise comparison semantics* ($\succ_{\mathcal{JP}}$) *satisfies Weak Defense Precedence.*

Proof. It suffices to show that given two vote arguments $\mathcal{A}, \mathcal{B} \in \mathbf{A}_{\mathcal{R}}$ s.t. $d(\mathcal{A}) \neq 0$ and $d(\mathcal{B}) \neq 0$, if $|Att_1(\mathcal{A})| = |Att_1(\mathcal{B})|, Def_2(\mathcal{A}) \neq \emptyset$ and $Def_2(\mathcal{B}) = \emptyset$ then $d(\mathcal{A}) > d(\mathcal{B})$.

Let $x = |Att_1(\mathcal{A})| = |Att_1(\mathcal{B})|$ and $|Def_2(\mathcal{A})| = y$, with $x, y > 0$. Note that $d(\mathcal{B}) \neq 0$ and $|Def_2(\mathcal{B})| = 0$. Hence, the degree of $\mathcal{B}$ is $d(\mathcal{B}) = w(\mathcal{B}) - ap(G_{\mathcal{B}})$, where $ap(G_{\mathcal{B}}) = x \cdot \frac{1}{\frac{m \cdot (m-1)}{2}}$. Also, note that $d(\mathcal{A}) \neq 0$, and hence the degree of $\mathcal{A}$ is $d(\mathcal{A}) = w(\mathcal{A}) - ap(G_{\mathcal{A}})$, where $ap(G_{\mathcal{A}}) = (x - y) \cdot \frac{1}{\frac{m \cdot (m-1)}{2}}$. From the above conditions we have that $ap(G_{\mathcal{B}}) > ap(G_{\mathcal{A}})$, and since $w(\mathcal{A}) = w(\mathcal{B}) = 1$, we conclude that $d(\mathcal{A}) > d(\mathcal{B})$. □

$\mathcal{VAF}$ Desirable Properties with respect to Social Choice

In this section we study the properties that should be satisfied by a Voting Argumentation Framework and its corresponding semantics from a social-choice theoretic scope.

Justified Preference Profile consistency with respect to AF. The first property refers to the relation the Voting argumentation framework should have with respect to the produced justified preference profile. We will show that the following property is satisfied when pairwise comparison semantics are applied to $\mathcal{VAF}$ for computing the justified preference profile.

Definition 10 (Justified Preference Profile consistency with respect to AF)**.** *If a unification of vote arguments $\mathbf{A}_{r_i}$ is stronger than $\mathbf{A}_{r_j}$, i.e., the degree of $\mathbf{A}_{r_i}$ is higher than $\mathbf{A}_{r_j}$, then the corresponding total order r_i should appear more times than the total order r_j in the justified preference profile.*

Example 1 (cont.: Justified Preference Profile consistency with respect to AF)**.** *Assume that another vote argument $\mathcal{A}_{r_{1,v_4}}$ is added to the previous voting argumentation framework. Figure 2.4 presents this new $\mathcal{VAF}$.*

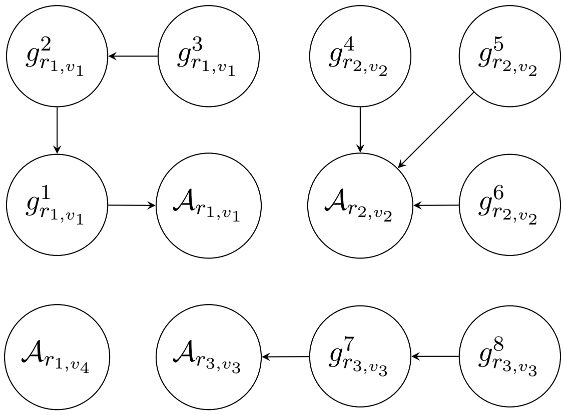


Figure 2.4: $\mathcal{VAF}$ exemplifying the Justified Preference Profile consistency with respect to AF property.

We can now recompute the justified preference profile $\succ_{\mathcal{JP}}$ the same way as before. We have that $d(\mathbf{A}_{r_1}) = d(\mathcal{A}_{r_{1,v_1}}) + d(\mathcal{A}_{r_{1,v_4}}) = \frac{2}{3} + 1 = \frac{5}{3}, d(\mathbf{A}_{r_2}) = 0, d(\mathbf{A}_{r_3}) = 1$. Therefore, $\sum_{\forall r_i} uv(r_i) = \frac{5}{3} + 1 = \frac{8}{3}$ and $gcd(1, 0, \frac{5}{3}) = \frac{1}{3}$. The total number of agents n' in $\succ_{\mathcal{JP}}$ is $n' = \frac{8/3}{1/3} = 8$. Therefore,

$$\begin{aligned} |r_1|_{\succ_{\mathcal{JP}}} &= 8 \cdot \frac{5/3}{8/3} = 5 \\ |r_2|_{\succ_{\mathcal{JP}}} &= 8 \cdot \frac{0}{8/3} = 0 \\ |r_3|_{\succ_{\mathcal{JP}}} &= 8 \cdot \frac{1}{8/3} = 3 \end{aligned}$$

Hence, now r_1 is the ranking with the highest degree and it appears more times in $\succ_{\mathcal{JP}}$ so the property is satisfied.

Theorem 3. *The justified preference profile computed by $\mathcal{VAF}$ under pairwise comparison semantics ($\succ_{\mathcal{JP}}$) satisfies Justified Preference Profile consistency with respect to AF.*

Proof. Let us suppose that a unification of vote arguments $\mathbf{A}_{r_i}$ is stronger than $\mathbf{A}_{r_j}$, and hence the degree $d(\mathbf{A}_{r_i}) > d(\mathbf{A}_{r_j})$ or $uv(r_i) > uv(r_j)$. The corresponding total order r_i appears in the $\succ_{\mathcal{JP}}$, $|r_i|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r} uv(r)}$ times while $|r_j|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_j)}{\sum_{\forall r} uv(r)}$. Given that $uv(r_i) > uv(r_j)$, we conclude that $|r_i|_{\succ_{\mathcal{JP}}} > |r_j|_{\succ_{\mathcal{JP}}}$. □

Social choice profile consistency The second property that we are going to check is a new but fundamental property that quantitative voting argumentation frameworks should have. We call it the social choice profile consistency. The intuition behind this property is that if there is no deliberation phase for the preferences of the agents then the outcome of the Voting Argumentation Framework, i.e., the justified preference profile should be the same as the outcome of the original social choice profile, since there is no new information or conflicts expressed by arguments. More formally, this property can be defined as follows.

Definition 11 (Social choice profile consistency)**.** *The semantics of a Voting Argumentation Framework satisfies* Social choice profile consistency *if when there is no deliberation phase, i.e., no generic arguments attacking the vote arguments, then the justified preference profile should be the same as the original social choice profile.*

Theorem 4. *The justified preference profile computed by $\mathcal{VAF}$ under pairwise comparison semantics ($\succ_{\mathcal{JP}}$) satisfies social choice profile consistency.*

Proof. Let $\succ$ be the social choice preference profile and $\succ_{\mathcal{JP}}$ the justified preference profile for the $\mathcal{VAF}$ when the pairwise comparison semantics is applied. In order to prove that $\mathcal{VAF}$ satisfies this property we have to show that each total order $r_i \in \succ$ is also included is $\succ_{\mathcal{JP}}$ the same number of times.

Let r_i be a given ranking included in $\succ$. It suffices to show that $|r_i|_{\succ} = |r_i|_{\succ_{\mathcal{JP}}}$, where $|r_i|_{\succ}$ is the number of times r_i appears in $\succ$ and $|r_i|_{\succ_{\mathcal{JP}}}$ the number of times it appears in $\succ_{\mathcal{JP}}$.

Recall that

$$|r_i|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}$$

Also, recall that $uv(r_i) = \sum_{\forall j} d(\mathcal{A}_{r_{i,j}})$ and $d(\mathcal{A}_{r_{i,j}}) = w(\mathcal{A}_{r_{i,j}}) - ap(G_{r_{i,j}})$.

Note that $|r_i|_{\succ} = \sum_{\forall j} w(\mathcal{A}_{r_{i,j}})$ under pairwise comparison semantics and that $\sum_{\forall j} ap(G_{r_{i,j}}) = 0$ since there are no attacks on the preferences. Hence, $uv(r_i) = |r_i|_{\succ}$.

The number of agents n' in the $\succ_{\mathcal{JP}}$ is $\sum_{\forall r_i} uv(r_i)$ since $uv(r_i)$ equals to $|r_i|_{\succ}$ and therefore each $uv(r_i)$ belongs to the set of natural numbers $\mathcal{N}$. Hence,

$$|r_i|_{\succ_{\mathcal{JP}}} = \frac{|r_i|_{\succ}}{\sum_{\forall r_i} uv(r_i)} \cdot \sum_{\forall r_i} uv(r_i) = |r_i|_{\succ}$$

□

Measuring the effect of deliberation on avoiding voting cycles In this section we are going to study the impact of a deliberation phase in social choice theory and especially on voting cycles, i.e, the Condorcet Paradox. The intuition that triggered this study is that if there exists an amount of ambiguous information in the preferences of the agents the deliberation phase can reveal it using the imposed arguments and "correct" the misinformation on the preferences of the agents that trigger voting cycles. In order to study this impact we are measuring the effect of a deliberation phase in producing a justified preference profile where there are no voting cycles. The metric that we are going to use to measure the impact is the number of arguments needed to attack preferences of the original profile so that no cycles exist. In other words, it is the attacking power of the generic arguments for total order r_i, i.e., $ap(G_{r_i})$.

Theorem 5. *Given that for alternatives $a, b, c \in A$, a beats b and b beats c in both $\succ$ and $\succ_{\mathcal{JP}}$, the justified preference profile computed by $\mathcal{VAF}$ does not produce any voting cycle when the following conditions on the attacking power of the generic arguments hold.*

1. $\sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}) \leq \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}})$, *when* $|\mathcal{R}_{ac}|_{\succ} > |\mathcal{R}_{ca}|_{\succ}$
2. $\sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}) > \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}})$,
   when $|\mathcal{R}_{ac}|_{\succ} - |\mathcal{R}_{ca}|_{\succ} > \sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}) - \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}})$
3. $\sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}) < \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}})$,
   when $|\mathcal{R}_{ac}|_{\succ} < |\mathcal{R}_{ca}|_{\succ} < \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}}) - \sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}) + |\mathcal{R}_{ac}|_{\succ}$

The above relations state that in order to avoid the cycle we must have, for 3 alternatives $a, b, c \in A$ where a beats b and b beats c in $\succ$ and $\succ_{\mathcal{JP}}$, the following conditions:

1. The attacking power for the vote arguments $\mathbf{A}_{\mathcal{R}_{ac}}$ is less or equal to the attacking power for the vote arguments $\mathbf{A}_{\mathcal{R}_{ca}}$, when the majority of agents prefer a over c in the preference profile $\succ$.

2. The attacking power for the vote arguments $\mathbf{A}_{\mathcal{R}_{ac}}$ is greater than the attacking power for the vote arguments $\mathbf{A}_{\mathcal{R}_{ca}}$, when the difference in number of agents preferring $a \succ c$ to $c \succ a$ is lower bounded by the difference of the corresponding attacking powers.

3. The attacking power for the vote arguments $\mathbf{A}_{\mathcal{R}_{ac}}$ is less than the attacking power for the vote arguments $\mathbf{A}_{\mathcal{R}_{ca}}$, when the majority of agents prefer c over a in the preference profile $\succ$ and the difference in number of agents preferring $c \succ a$ to $a \succ c$ is upper bounded by the difference of the corresponding attacking powers.

Proof. Let $\mathcal{R}_{xy}$ be the subset of the total orders set $\mathcal{R} \in \succ_{\mathcal{JP}}$ in which x is ranked over y. Hence, all the possible sets of vote arguments which are derived from $\mathcal{R}$ between 3 alternatives a, b, c are the following: $\mathbf{A}_{\mathcal{R}_{ab}}$, $\mathbf{A}_{\mathcal{R}_{ba}}$, $\mathbf{A}_{\mathcal{R}_{bc}}$,$\mathbf{A}_{\mathcal{R}_{cb}}$,$\mathbf{A}_{\mathcal{R}_{ac}}$, $\mathbf{A}_{\mathcal{R}_{ca}}$.

Recall that the number of rankings r_i in $\succ_{\mathcal{JP}}$ is given by $|r_i|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}$, which gives us that $|r_i|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{|r_i|_{\succ} - ap(G_{r_i})}{n - ap(G)}$, where n is the number of agents in $\succ$.

Let r_{xy} be a total order where x is ranked over y. Summing up for all $r_{xy} \in \mathcal{R}_{xy}$ we have that $|\mathcal{R}_{xy}|_{\succ_{\mathcal{JP}}} = \sum_{r_{xy} \in \mathcal{R}_{xy}} |r_{xy}|_{\succ_{\mathcal{JP}}}$. Hence, $|\mathcal{R}_{xy}|_{\succ_{\mathcal{JP}}} = \frac{n\prime}{n - ap(G)} \cdot (\sum_{r_{xy} \in \mathcal{R}_{xy}} |r_{xy}|_{\succ} - \sum_{r_{xy} \in \mathcal{R}_{xy}} ap(G_{r_{xy}}))$.

In order to have a voting cycle there must exist a justified preference profile $\succ_{\mathcal{JP}}$ where there are 3 alternatives a, b, c such that a is ranked over b, b is ranked over c and c is ranked over a, in at least $\frac{n\prime}{2}$ agents, where $n\prime$ is the number of agents in $\succ_{\mathcal{JP}}$.

Therefore, in order to avoid a cycle in $\succ_{\mathcal{JP}}$ then for all $a, b, c \in A$ if a beats b and b beats c in pairwise comparisons then a should beat also c. Therefore, we have that

$$\begin{cases} |\mathcal{R}_{ab}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{ba}|_{\succ_{\mathcal{JP}}}, \\ |\mathcal{R}_{bc}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{cb}|_{\succ_{\mathcal{JP}}}, \\ |\mathcal{R}_{ac}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{ca}|_{\succ_{\mathcal{JP}}} \end{cases}$$

This means that given that $|\mathcal{R}_{ab}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{ba}|_{\succ_{\mathcal{JP}}}, |\mathcal{R}_{bc}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{cb}|_{\succ_{\mathcal{JP}}}$ then in order to avoid cycle it must be that $|\mathcal{R}_{ac}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{ca}|_{\succ_{\mathcal{JP}}}$. Since, $|\mathcal{R}_{ac}|_{\succ_{\mathcal{JP}}} = \frac{n\prime}{n - ap(G)} \cdot (\sum_{r_{ac} \in \mathcal{R}_{ac}} |r_{ac}|_{\succ} - \sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}))$ and $|\mathcal{R}_{ca}|_{\succ_{\mathcal{JP}}} = \frac{n\prime}{n - ap(G)} \cdot (\sum_{r_{ca} \in \mathcal{R}_{ca}} |r_{ca}|_{\succ} - \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}}))$ we have that

$$\begin{aligned} &\sum_{r_{ac} \in \mathcal{R}_{ac}} |r_{ac}|_{\succ} - \sum_{r_{ac} \in \mathcal{R}_{ac}} ap(G_{r_{ac}}) \\ &> \sum_{r_{ca} \in \mathcal{R}_{ca}} |r_{ca}|_{\succ} - \sum_{r_{ca} \in \mathcal{R}_{ca}} ap(G_{r_{ca}}) \end{aligned} \tag{2.4}$$

We distinguish between two cases for the original preference profile $\succ$. The first case is when there is no cycle in $\succ$. Then for all $a, b, c \in A$ and if a beats b and b beats c in pairwise comparisons then a beats also c. Hence, we have that given

$$\begin{cases} |\mathcal{R}_{ab}|_{\succ} > |\mathcal{R}_{ba}|_{\succ}, \\ |\mathcal{R}_{bc}|_{\succ} > |\mathcal{R}_{cb}|_{\succ} \end{cases}$$

then $|\mathcal{R}_{ac}|_{\succ} > |\mathcal{R}_{ca}|_{\succ}$.

From the last inequality and inequality 2.4 we have that given that a beats b and b beats c

in $\succ$ and $\succ_{\mathcal{JP}}$ then

$$\begin{aligned}
\sum_{r_{ac}\in\mathcal{R}_{ac}} ap(G_{r_{ac}}) &\leq \sum_{r_{ca}\in\mathcal{R}_{ca}} ap(G_{r_{ca}}), \text{ when} \\
|\mathcal{R}_{ac}|_{\succ} &> |\mathcal{R}_{ca}|_{\succ} \\
\\
\sum_{r_{ac}\in\mathcal{R}_{ac}} ap(G_{r_{ac}}) &> \sum_{r_{ca}\in\mathcal{R}_{ca}} ap(G_{r_{ca}}), \text{ when} \\
|\mathcal{R}_{ac}|_{\succ} - |\mathcal{R}_{ca}|_{\succ} &> \sum_{r_{ac}\in\mathcal{R}_{ac}} ap(G_{r_{ac}}) \\
&\quad - \sum_{r_{ca}\in\mathcal{R}_{ca}} ap(G_{r_{ca}})
\end{aligned} \tag{2.5}$$

The second case is when there is a cycle in $\succ$. Then there exist $a, b, c \in A$ such that

$$\begin{cases} |\mathcal{R}_{ab}|_{\succ} > |\mathcal{R}_{ba}|_{\succ}, \\ |\mathcal{R}_{bc}|_{\succ} > |\mathcal{R}_{cb}|_{\succ}, \\ |\mathcal{R}_{ca}|_{\succ} > |\mathcal{R}_{ac}|_{\succ} \end{cases}$$

From the last inequality and inequality 2.4 we have that given that a beats b and b beats c in $\succ$ and $\succ_{\mathcal{JP}}$ then

$$\begin{aligned}
\sum_{r_{ac}\in\mathcal{R}_{ac}} ap(G_{r_{ac}}) &< \sum_{r_{ca}\in\mathcal{R}_{ca}} ap(G_{r_{ca}}), \text{ when} \\
|\mathcal{R}_{ac}|_{\succ} < |\mathcal{R}_{ca}|_{\succ} &< \sum_{r_{ca}\in\mathcal{R}_{ca}} ap(G_{r_{ca}}) \\
&\quad - \sum_{r_{ac}\in\mathcal{R}_{ac}} ap(G_{r_{ac}}) + |\mathcal{R}_{ac}|_{\succ}
\end{aligned} \tag{2.6}$$

Hence, by summing up conditions 2.5 and 2.6 we infer that a voting cycle can be avoided when the abovementioned relations on attacking powers exist and the proof is complete.

□

$\mathcal{VAF}$ attitude towards classical Social Choice desirable properties

In this section we explore the behaviour of the proposed approach towards classical desirable properties from the viewpoint of social choice. Apart from the properties that should be satisfied by a Voting Argumentation Framework and its corresponding semantics which are related to the social choice outcome, we are also referring to classical desirable properties in order to further evaluate $\mathcal{VAF}$ and pairwise semantics. We begin by studying the properties of homogeneity and monotonicity which both belong to the stability category as recognized by social choice theorists. The properties of this category are concerned with ensuring that the winning set remains the same when the changes in the preference profile arguably should not modify the winning set. Our goal here is to evaluate the effect of deliberation and argumentation in voting and investigate how the consideration of the deliberation phase along with voting can affect the outcome in respect with the social choice properties.

Homogeneity The first property discussed in this section is the one of homogeneity. A method is homogeneous if the replication[7] of the preference profile does not change the winning set of the alternatives. We prove the following positive result for our proposed method which ensures that replicating the original instance of the problem, i.e., the agents' preferences and the justifications, will have no effect on the decision output as long as the voting rule used for the aggregation of the justified preference profile satisfies homogeneity. This property is widely accepted among social choice theorists since making an *exact copy* of the agents' preferences x times should not have an effect on the winning set.

Example 1 (cont.: Homogeneity)**.** *Assume that the original preference profile along with its arguments is replicated* 2 *times. That means that now we have added the following vote arguments* $\mathcal{A}'_{r_{1,v_1}}, \mathcal{A}'_{r_{2,v_2}}, \mathcal{A}'_{r_{3,v_3}}, \mathcal{A}'_{r_{1,v_4}}$.

We can now recompute the new justified preference profile $\succ'_{\mathcal{JP}}$ *the same way as before. We have that* $d(\mathbf{A}_{r_1}) = d(\mathcal{A}_{r_{1,v_1}}) + d(\mathcal{A}_{r_{1,v_4}}) + d(\mathcal{A}'_{r_{1,v_1}}) + d(\mathcal{A}'_{r_{1,v_4}}) = \frac{2}{3} + 1 + \frac{2}{3} + 1 = \frac{10}{3}, d(\mathbf{A}_{r_2}) = 0, d(\mathbf{A}_{r_3}) = 2$. *Therefore,* $\sum_{\forall r_i} uv(r_i)' = \frac{10}{3} + 2 = \frac{16}{3}$ *and* $gcd(2, 0, \frac{10}{3}) = \frac{2}{3}$. *The total number of agents in* $\succ'_{\mathcal{JP}}$ *is* $n'_{\succ'_{\mathcal{JP}}} = \frac{16/3}{2/3} = 8$. *Therefore,* $|r_1|_{\succ_{\mathcal{JP}}}$ *is 5,* $|r_2|_{\succ_{\mathcal{JP}}}$ *is 0 and* $|r_3|_{\succ_{\mathcal{JP}}}$ *is 3. Hence, the justified replicated profile is the same as the original justified profile. Therefore an application of a voting rule on both profiles will have the exact same winning ranking and the property is satisfied.*

Theorem 6. $\mathcal{VAF}$ *satisfies homogeneity if the voting rule used for the aggregation of* $\succ_{\mathcal{JP}}$ *satisfies also homogeneity.*

Proof. Suppose that $\mathcal{R}$ is the winning ranking after applying $\mathcal{VAF}$ and pairwise comparison semantics in the original instance of the problem, i.e., having as input the preference profile $\succ$ and the generic arguments G. If we replicate the original profile by x times then we get as input the new profile $\succ^{(x)}$ and the set of generic arguments $G^{(x)}$. For each $r_i \in \succ$ we have that $|r_i|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}$ and also for each $r_i \in \succ^{(x)}$ we have that $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = n^{('x)} \cdot \frac{uv(r_i)^{(x)}}{\sum_{\forall r_i} uv(r_i)^{(x)}}$. We compute the utility value for r_i, i.e., $uv(r_i)^{(x)}$, for the replicated justified profile $\succ^{(x)}$. We have that $uv(r_i)^{(x)} = |r_i|^{(x)}_{\succ} - ap^{(x)}(G_{r_i})$, where $|r_i|^{(x)}_{\succ}$ is the number of times that r_i appears in the replicated profile and $ap^{(x)}(G_{r_i})$ the attacking power for r_i in the replicated profile. Since the profile and the generic arguments are replicated x times we have that $|r_i|^{(x)}_{\succ} = x \cdot |r_i|_{\succ}$ and $ap^{(x)}(G_{r_i}) = x \cdot ap(G_{r_i})$. Hence,

$$uv(r_i)^{(x)} = x \cdot |r_i|_{\succ} - x \cdot ap(G_{r_i}) = x \cdot uv(r_i) \tag{2.7}$$

[7]Please note that, in our context, the replication concerns both the preference profile and the arguments referring to this preference profile.

The number of agents both in the original and the replicated justified profile vary according to $uv(r_i)$ so we distinguish between the following cases.

Case 1: $uv(r_i) \in \mathcal{N} \Rightarrow uv(r_i)^{(x)} \in \mathcal{N}$. This means that $n' = \sum_{\forall r_i} uv(r_i)$ and $|r_i|_{\succ_{\mathcal{JP}}} = \sum_{\forall r_i} uv(r_i) \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)} = uv(r_i)$.

We have that $n^{('x)} = \sum_{\forall r_i} uv(r_i)^{(x)}$ and $\sum_{\forall r_i} uv(r_i)^{(x)} = \sum uv(r_i)^{(x)}$ which gives us from Equation 2.7 that $\sum_{\forall r_i} uv(r_i)^{(x)} = x \cdot \sum_{\forall r_i} uv(r_i)$. Hence, $n^{('x)} = x \cdot \sum_{\forall r_i} uv(r_i)$.

Now, we compute $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = (x \cdot \sum_{\forall r_i} uv(r_i)) \cdot \frac{uv(r_i)^{(x)}}{x \cdot \sum_{\forall r_i} uv(r_i)} = x \cdot uv(r_i)$. The last equality holds from Equation 2.7 and so $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = x \cdot |r_i|_{\succ_{\mathcal{JP}}}$. This means that each vote of the original justified profile is replicated x times in the replicated justified profile. Hence, if the voting rule applied to the replicated profile satisfies homogeneity then also $\mathcal{VAF}$ does.

Case 2: $uv(r_i) \in \mathcal{Q} \setminus \mathcal{N}$ and $uv(r_i)^{(x)} \in \mathcal{N}$. This means that $n' = \frac{\sum_{\forall r_i} uv(r_i)}{gcd(uv(r_i), \forall r_i \in N')}$ and $|r_i|_{\succ_{\mathcal{JP}}} = \frac{\sum_{\forall r_i} uv(r_i)}{gcd(uv(r_i), \forall r_i \in N')} \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}$. The last equation gives us that

$$uv(r_i) = gcd(uv(r_i), \forall r_i \in N') \cdot |r_i|_{\succ_{\mathcal{JP}}} \tag{2.8}$$

Since $uv(r_i)^{(x)} \in \mathcal{N}$ we have that $n^{('x)} = \sum_{\forall r_i} uv(r_i)^{(x)}$.

Now, we compute $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = \sum_{\forall r_i} uv(r_i)^{(x)} \cdot \frac{uv(r_i)^{(x)}}{\sum_{\forall r_i} uv(r_i)^{(x)}} = x \cdot uv(r_i)$. The last equality holds from Equation 2.7 and so $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = x \cdot |r_i|_{\succ_{\mathcal{JP}}}$. From Equation 2.8 we have that $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = x \cdot gcd(uv(r_i), \forall r_i \in N') \cdot |r_i|_{\succ_{\mathcal{JP}}}$. This gives us that $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = w \cdot |r_i|_{\succ_{\mathcal{JP}}}$, where $w = x \cdot gcd(uv(r_i), \forall r_i \in N')$ and hence, $w \in \mathcal{N}$. So, in this case also each vote of the original justified profile is replicated w times in the replicated justified profile. Hence, if the voting rule applied to the replicated profile satisfies homogeneity then also $\mathcal{VAF}$ does.

Case 3: $uv(r_i) \in \mathcal{Q} \setminus \mathcal{N}$ and $uv(r_i)^{(x)} \in \mathcal{Q} \setminus \mathcal{N}$. This means that $n' = \frac{\sum_{\forall r_i} uv(r_i)}{gcd(uv(r_i), \forall r_i \in N')}$ and $|r_i|_{\succ_{\mathcal{JP}}} = \frac{\sum_{\forall r_i} uv(r_i)}{gcd(uv(r_i), \forall r_i \in N')} \cdot \frac{uv(r_i)}{\sum_{\forall r_i} uv(r_i)}$ which gives also Equation 2.8.

Since $uv(r_i)^{(x)} \in \mathcal{Q} \setminus \mathcal{N}$ we have that $n^{('x)} = \frac{\sum_{\forall r_i} uv(r_i)^{(x)}}{gcd(uv(r_i)^{(x)}, \forall r_i \in N'^{(x)})}$. From Equation 2.7 we have that $n^{('x)} = \frac{\sum_{\forall r_i} uv(r_i)^{(x)}}{gcd(x \cdot uv(r_i), \forall r_i \in N'^{(x)})} = \frac{\sum_{\forall r_i} uv(r_i)^{(x)}}{x \cdot gcd(uv(r_i), \forall r_i \in N'^{(x)})}$. Now, we compute $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = \frac{\sum_{\forall r_i} uv(r_i)^{(x)}}{x \cdot gcd(uv(r_i), \forall r_i \in N'^{(x)})} \cdot \frac{uv(r_i)^{(x)}}{\sum_{\forall r_i} uv(r_i)^{(x)}} = \frac{x \cdot uv(r_i)}{x \cdot gcd(uv(r_i), \forall r_i \in N'^{(x)})}$. The last equality holds from Equation 2.7 and so given Equation 2.8 we have that $|r_i|^{(x)}_{\succ_{\mathcal{JP}}} = |r_i|_{\succ_{\mathcal{JP}}}$. Hence, in this case the original justified profile is identical to the replicated one and therefore homogeneity is satisfied for every voting rule applied to $\mathcal{VAF}$ since any voting rule applied to the same input profile gives the same winning ranking. □

Monotonicity The second property discussed in this section is the one of monotonicity. A method is monotonic if a winning alternative remains the winning one in the new profile which is created after she is moved upward in the preferences of some of the agents. We prove the following positive result for our proposed method which ensures that if an alternative who is a winner in an instance of the problem is moved upwards in the preferences of some agents then this will have no effect on the decision output of the new instance as long as (1) the voting rule used for the aggregation of the justified preference profile satisfies monotonicity, (2) the attacking power on the votes where the winner has a better position (compared to the original profile) is equal to the attacking power on the corresponding votes in the original profile. This property is also widely accepted among social choice theorists since improving the position of a winner alternative in some agents should not make him worse and should remain the winner.

Theorem 7. *$\mathcal{VAF}$ under pairwise semantics satisfies monotonicity if the voting rule used for the aggregation of $\succ_{\mathcal{JP}}$ satisfies monotonicity and the attacking power of the w-improvement votes is equal to the attacking power of the votes that change in the original preference profile.*

Proof. Suppose that w is the winning alternative in the original instance of the problem, i.e., having as input the preference profile $\succ$. If we raise w in the preferences of some agents then we get as input the new profile $\succ'$. Let R_u the set of agents votes that remain the same in both profiles and R_c the set of agents votes in $\succ$ that change. We denote the set of changed votes in $\succ'$ as R'_c. The votes $r'_c \in R'_c$ are w-improvements of the $r_c \in R_c$, i.e., w is ranked higher in $r'_c \in R'_c$ compared to $r_c \in R_c$. Since the voting rule used in the $\mathcal{VAF}$ satisfies monotonicity then it suffices to show that $\succ'_{\mathcal{JP}}$, i.e, the justified profile in the new instance is a w-improvement of $\succ_{\mathcal{JP}}$, i.e., the justified profile in the original instance. In order for $\succ'_{\mathcal{JP}}$ to be a w-improvement of $\succ_{\mathcal{JP}}$ then the following conditions must exist: $\forall r_u \in \succ_{\mathcal{JP}} \Rightarrow r_u \in \succ'_{\mathcal{JP}}, \forall r_c \in \succ_{\mathcal{JP}} \Rightarrow r'_c \in \succ'_{\mathcal{JP}}, n_{\mathcal{JP}} = n'_{\mathcal{JP}}$. With $n_{\mathcal{JP}}$ we denote the number of agents in $\succ_{\mathcal{JP}}$ and with $n'_{\mathcal{JP}}$ the number of agents in $\succ'_{\mathcal{JP}}$.

In order to fulfill the first condition each r_u in $\succ_{\mathcal{JP}}$ must remain also in the $\succ'_{\mathcal{JP}}$ and hence, $|r_u|_{\succ_{\mathcal{JP}}} = |r_u|_{\succ'_{\mathcal{JP}}}$. We have that the number of total orders $|r_u|_{\succ_{\mathcal{JP}}} = n_{\mathcal{JP}} \cdot \frac{|r_u|_{\succ} - ap(G_{r_u})}{n - ap(G)}$, where n the number of agents in $\succ$. In the new instance, we have that $|r_u|_{\succ'_{\mathcal{JP}}} = n'_{\mathcal{JP}} \cdot \frac{|r_u|_{\succ'} - ap'(G_{r_u})}{n' - ap'(G)}$. Note that $n_{\mathcal{JP}} = n'_{\mathcal{JP}}$ from the above conditions. Also note that $|r_u|_{\succ'} = |r_u|_{\succ}$ and $ap(G_{r_u}) = ap'(G_{r_u})$, since the unchanged votes and the generic arguments that refer to them are the same in both instances. Finally, note that $n = n'$ since the number of agents in both profiles $\succ$ and $\succ'$ is the same. Therefore, given $|r_u|_{\succ_{\mathcal{JP}}} = |r_u|_{\succ'_{\mathcal{JP}}}$ we have that $ap(G) = ap'(G)$.

In order to fulfill the second condition each $r'_c \in \succ'_{\mathcal{JP}}$ must be a w-improvement of r_c in $\succ_{\mathcal{JP}}$ and hence, $|r_c|_{\succ_{\mathcal{JP}}} = |r'_c|_{\succ'_{\mathcal{JP}}}$. We have that the number of total orders $|r_c|_{\succ_{\mathcal{JP}}} = n_{\mathcal{JP}} \cdot \frac{|r_c|_{\succ} - ap(G_{r_c})}{n - ap(G)}$, where n the number of agents in $\succ$. In the new instance, we have that $|r'_c|_{\succ'_{\mathcal{JP}}} = n'_{\mathcal{JP}} \cdot \frac{|r'_c|_{\succ'} - ap'(G_{r'_c})}{n' - ap'(G)}$. Note that $n_{\mathcal{JP}} = n'_{\mathcal{JP}}$ from the above conditions. Also note that $|r'_c|_{\succ'} = |r_c|_{\succ}$, since the changed votes and their w-improvements are the same in both instances of the preference profile $\succ$. Finally, note that $n = n'$ since the number of agents in both profiles $\succ$ and $\succ'$ is the same. Therefore, given $|r_c|_{\succ_{\mathcal{JP}}} = |r'_c|_{\succ'_{\mathcal{JP}}}$ we have $ap(G_{r_c}) = ap'(G_{r'_c})$. If the last equality holds and since $ap(G_{r_u}) = ap'(G_{r_u})$ that means that $ap(G) = ap'(G)$ and the theorem holds.

□

Consistency related to Majority In this section, we present our results about classical properties coming from social choice theory which in general concern the consistency between the outcomes of the given method and the majority rule.

Condorcet consistency. One of the most meaningful properties is the Condorcet consistency. This property states that a method $\mathcal{F}$ satisfies Condorcet consistency if whenever there is an alternative c who beats every other alternative in a pairwise comparison (i.e., c is the dominant alternative) then c is the winner under $\mathcal{F}$.

Theorem 8. *$\mathcal{VAF}$ under pairwise semantics satisfies Condorcet consistency if the voting rule used for the aggregation of $\succ_{\mathcal{JP}}$ satisfies Condorcet consistency and the following condition holds: for any alternatives $a, c \in A$*

$$|\mathcal{R}_{ca}|_{\succ} - |\mathcal{R}_{ac}|_{\succ} > ap(G_{\mathcal{R}_{ca}}) - ap(G_{\mathcal{R}_{ac}})$$

where $\mathcal{R}_{xy}$ denotes the number or agents that rank alternative x over y and $ap(G_{\mathcal{R}_{xy}})$ denotes the attacking power of the generic arguments on the arguments of the agents that rank x over y and c is the dominant alternative.

The theorem states that Condorcet consistency is satisfied if the difference in the number of agents preferring the dominant alternative c to every other alternative a minus the number of agents preferring a to c is greater than the difference of the corresponding attacking power on the vote arguments.

Proof. Let $c \in A$ be the dominant alternative and r_{ca} a ranking where c is over a, $a \in A$. We denote with the set $\mathcal{R}_{ca}$ the set of all the total orders r_{ca}. Since c is dominant $|\mathcal{R}_{ca}|_{\succ} > |\mathcal{R}_{ac}|_{\succ}$ for all $a \in A$. We have that $|r_{ca}|_{\succ_{\mathcal{JP}}} = n' \cdot \frac{uv(r_{ca})}{\sum_{\forall r_i} uv(r_i)}, a \in A$. Hence,

$$|\mathcal{R}_{ca}|_{\succ_{\mathcal{JP}}} = \sum_{\forall r_{ca}} |r_{ca}|_{\succ_{\mathcal{JP}}} = \frac{n'}{\sum_{\forall r_i} uv(r_i)} \cdot \sum_{\forall r_{ca}} uv(r_{ca}) \tag{2.9}$$

The condition for c to be the winner under $\mathcal{VAF}$ and $\succ_{\mathcal{JP}}$ (and hence, to satisfy Condorcet consistency) is that c must be dominant in $\succ_{\mathcal{JP}}$ since the voting rule used for the aggregation of $\succ_{\mathcal{JP}}$ satisfies the Condorcet consistency. Therefore, $|\mathcal{R}_{ca}|_{\succ_{\mathcal{JP}}} > |\mathcal{R}_{ac}|_{\succ_{\mathcal{JP}}}, \forall a \in A$.

Given Equation 2.9 and the last inequality, we have:

$$\sum_{\forall r_{ca}} uv(r_{ca}) > \sum_{\forall r_{ac}} uv(r_{ac})$$
$$\Rightarrow$$
$$\sum_{\forall r_{ca}} |r_{ca}|_{\succ} - \sum_{\forall r_{ca}} ap(G_{r_{ca}}) > \sum_{\forall r_{ac}} |r_{ac}|_{\succ} - \sum_{\forall r_{ac}} ap(G_{r_{ac}})$$
$$\Rightarrow$$
$$|\mathcal{R}_{ca}|_{\succ} - ap(G_{\mathcal{R}_{ca}}) > |\mathcal{R}_{ac}|_{\succ} - ap(G_{\mathcal{R}_{ac}})$$

This gives us the condition that

$$|\mathcal{R}_{ca}|_{\succ} - |\mathcal{R}_{ac}|_{\succ} > ap(G_{\mathcal{R}_{ca}}) - ap(G_{\mathcal{R}_{ac}})$$

□

Invariant loss consistency. This property is also based on the Condorcet's intuition and is similar to the Condorcet consistency. It states that a method $\mathcal{F}$ satisfies Invariant loss consistency if whenever there is an alternative c who is beaten by every other alternative in a pairwise comparison then c cannot be the winner under $\mathcal{F}$.

Theorem 9. *$\mathcal{VAF}$ under pairwise semantics satisfies Invariant loss consistency if the voting rule used for the aggregation of $\succ_{\mathcal{JP}}$ satisfies Invariant loss consistency and following condition holds: for any alternatives $a, c \in A$*

$$|\mathcal{R}_{ca}|_{\succ} - |\mathcal{R}_{ac}|_{\succ} < ap(G_{\mathcal{R}_{ca}}) - ap(G_{\mathcal{R}_{ac}})$$

where $\mathcal{R}_{xy}$ denotes the number or agents alternative x over y and $ap(G_{\mathcal{R}_{xy}})$ denotes the attacking power of the generic arguments on the arguments of the agents that rank x over y and c is the alternative who is beaten by every other alternative (i.e, the Condorcet loser).

Proof. The proof is similar to the one above regarding the Condorcet consistency. Let $c \in A$ be the alternative who is beaten by every other alternative. Hence, $|\mathcal{R}_{ca}|_{\succ} < |\mathcal{R}_{ac}|_{\succ}$ for all $a \in A$.

The condition for c not to be in the winning set under $\mathcal{VAF}$ and $\succ_{\mathcal{JP}}$ (and hence, satisfy Invariant loss consistency) is that c must be beaten by every other alternative in $\succ_{\mathcal{JP}}$ since the voting rule used for the aggregation of $\succ_{\mathcal{JP}}$ satisfies the Invariant loss consistency. Therefore, $|\mathcal{R}_{ca}|_{\succ_{\mathcal{JP}}} < |\mathcal{R}_{ac}|_{\succ_{\mathcal{JP}}}, \forall a \in A$.

Following a similar approach as the proof of the above theorem gives us the condition that

$$|\mathcal{R}_{ca}|_{\succ} - |\mathcal{R}_{ac}|_{\succ} < ap(G_{\mathcal{R}_{ca}}) - ap(G_{\mathcal{R}_{ac}})$$

□

2.2.5 Conclusion and future work

In this paper, we have proposed a method for group decision-making that is built upon the justified preferences of the agents. Our method is able to simulate real decision problems where the decision outcome relies on agents' preferences and their reasoning. Our design choices assume and comply with a deliberation phase in which a discussion is conducted among the agents of the group so that the preferences along with justifications are revealed. In order to do so, we introduced a voting argumentation framework and its corresponding semantics. We proposed a method for computing a new preference profile based on the acceptability semantics of the vote arguments. We proved several properties from an argumentative and social choice theoretic point of view on the so-called justified preference profile.

In terms of future work, we plan to extend our research towards the proposed modelling and characterize other properties of social choice and argumentation that are satisfied by $\mathcal{VAF}$. Also, another future step is to define another kind of acceptability semantics and explore its properties. For example, it would be interesting to design semantics that permit us to avoid the Condorcet paradox under any case. Furthermore, it is also appealing to design different kinds of semantics which are specially adapted to distinctive real decision problems. Indeed, with the semantics proposed in this paper, the generic arguments have the same strength, but there are cases where generic arguments attacking vote arguments are not of equal importance. It is therefore a motivation to design a semantics with graded strength of generic arguments. It is also interesting to investigate more on the properties from an argumentative perspective (such as the ones defined in [29]) for the proposed semantics and identify which ones are respected. Finally, it is let for future work a practical application of this method in real decision problems. It will be very interesting to design a real experiment and see how the proposed method works by measuring the satisfaction of the agents for the decision outcome when their justifications are taken into account compared to unjustified preferences where pure social choice methods are used.

Chapter 3

Collective decision-making seen from a logic and knowledge base perspective

Multi-Criteria Decision Making with Existential Rules using Repair Techniques

In this chapter we study the problem of collective decision making under multiple criteria. In this problem, the agents express their preferences about the set of alternatives cardinally (rather than ordinally as in the problems studied so far). Also, here, the agents give a rating for each one of the criteria they consider for an alternative. Hence, we can formulate that agents express their preferences on alternatives over a set of multiple criteria. The objective is to reach an acceptable collective decision aggregating the agents' preferences. We take a new approach for this problem and examine it from a computational logic perspective. Therefore, we provide a novel modelling of the multi-criteria decision making problem as an inconsistent knowledge base, and we explain how to benefit from the reasoning capabilities of existential rules. The work following this approach has been presented in the "International Conference of the British Computer Society's Specialist Group on Artificial Intelligence (AI-2018)" and has been published in its proceedings [63].

Abstract

The central problem in multi-criteria decision making is to reach an acceptable decision aggregating preferences over multiple criteria. In this paper, we explain how to benefit from the reasoning capabilities of existential rules for modelling a multi-criteria decision making problem as an inconsistent knowledge base. The repairs of this knowledge base represent the maximally consistent point of views and inference strategies can be used for decision making.

3.1 Introduction

The way to reach a group's decision is a very complex task and depends on the nature of the decision problem. A common way for agents to take their decisions is to analyze the problem in different criteria which correspond to the different aspects that they consider to be important for the decision. This can be depicted as the multi-criteria decision making (MCDM) problem, where the set of agents corresponds to the decision makers that have preferences over multiple criteria on a set of alternatives. MCDM has been an active research area since the 60s when B. Roy ([19]) introduced the class of ELECTRE methods for aggregating preferences expressed on multiple criteria. Its development continued with the foundational "outranking methods" ([73]) and has been since an active research field. Recently these techniques have been used for recommender systems, whose objective is to recommend a solution given the decision makers' evaluation on different criteria of the alternative options. Many aggregation strategies for solving the group recommender systems can be applied to the multi-criteria ones [71].

In this paper we take a different research avenue and see the MCDM problem from a knowledge representation and reasoning point of view. The advantage of doing so is two fold. First, it allows for the use of expressive languages for describing the decision problem (and their subsequent reuse in applications). Second, it paves the way for synergies between the two fields. MCDM could thus benefit from recent advances on explanation and user interaction developed within the techniques we employ in this paper.

Concretely, we propose to see the problem description of a MCDM problem as an inconsistent knowledge base expressed using existential rules. The inconsistency will be addressed using repair techniques [14], a state of the art method of reasoning in presence of inconsistency that outputs the consistent subsets of the knowledge base maximal with respect to set inclusion. The reasoning is then performed on these subsets (also called repairs of the knowledge base).

Classically, in the MCDM problem, we have a set of alternative options (denoted by the set of constants $\mathcal{C}_A$) and a set of agents/decision makers $\mathcal{I}$. Each alternative expresses a set of alternatives (denoted by $\mathcal{C}_{A,\mathcal{I}}$) and a set of criteria (denoted by the set of predicates $\mathcal{P}_{C,\mathcal{I}}$). The set of all criteria predicates, i.e., the set of criteria given by all the decision makers is denoted by $\mathcal{P}_C$. The decision makers are called to express their preferences concerning each of these alternatives by taking into account the criteria.

We will use the knowledge bases produced by each decision maker in order to compute the set of repairs $\boldsymbol{E}$ which aggregates the different decision makers preferences. The inconsistency tolerant semantics will allow to reason on repairs. In this paper we do not focus on the inconsistency tolerant reasoning part but rather on the modelling as an inconsistent knowledge base. This is due to the fact that our contribution lays in the modelling aspects rather than reasoning aspects (that employ state of the art algorithms). Let us first present the logical language used throughout the paper.

3.2 Background notions

In this section we define the logical language employed in this paper. Please note that the notions below represent classical logical notions with some minor changes to reflect the decision making setting, in which we place ourselves in. We consider *the positive existential* fragment of first-order logic $\texttt{FOL}(\exists, \wedge)$ [38, 15]. Its language $\mathcal{L}$ is composed of formulas built with the usual quantifiers $(\exists, \forall)$ and *only* two connectors: implication $(\rightarrow)$ and conjunction $(\wedge)$.

We consider usual first-order vocabularies with constants but no other function symbols. A vocabulary is a pair of two disjoint sets $\mathcal{V} = (\mathcal{P}, \mathcal{C})$, where $\mathcal{P}$ is a finite set of predicates and $\mathcal{C}$ is a set of constants.

The following refining on the vocabulary definition is proper to the way we formalize the decision problem involving alternatives and values on the decision criteria over these alternatives. More precisely:

- The set of predicates $\mathcal{P}$ is partitioned in two disjoint sets $\mathcal{P} = \mathcal{P}_R \cup \mathcal{P}_C$ with $\mathcal{P}_R \cap \mathcal{P}_C = \emptyset$. $\mathcal{P}_C$ represents the criteria predicates we consider for decision making while $\mathcal{P}_R$ represents the other predicates (relations) which are used for describing the world in general (and thus not considered directly by the decision process).

- The set of constants $\mathcal{C}$ is partitioned into four pairwise disjoint sets $\mathcal{C} = \mathcal{C}_A \cup \mathcal{C}_\mathcal{I} \cup \mathcal{C}_V \cup \mathcal{C}_C$ and for all $x, y \in \{A, \mathcal{I}, V, C\}$ s.t. $x \neq y$, $\mathcal{C}_x \cap \mathcal{C}_y = \emptyset$. The set $\mathcal{C}_A$ is representing the constants that are naming the various alternatives involved in the decision process. The set $\mathcal{C}_\mathcal{I}$ represents the constants that are naming the agents/decision makers of $\mathcal{I}$ that are taking part in the decision-making problem. Let $\mathcal{C}_V = \bigcup_{c \in \mathcal{P}_C} \mathcal{C}_V^c$, where a set $\mathcal{C}_V^c$ represents the different values the criteria $c \in \mathcal{P}_C$ can have. Therefore the interpretation of a predicate c in $\mathcal{P}_C$ will only take values from $\mathcal{C}_A$ and their corresponding $\mathcal{C}_V^c$. Note that, the interpretation of an n-ary predicate symbol is a set of n-tuples of elements of the domain of discourse.The set $\mathcal{C}_C$ represents the other constants used eventually by $\mathcal{P}_R$. The domain of interpretation of $\mathcal{P}_R$ is $C_A \cup C_C$.

- Furthermore, for each predicate c in $\mathcal{P}_C$ we define on the values of its interpretation $C_A \times \mathcal{C}_V^{c\,(n-1)}$, where n is the arity of the predicate, a total order $\gg_{C_A \times \mathcal{C}_V^c}$. By abuse of notation, when the set of alternatives is given, we denote the preference $\gg_{\mathcal{C}_V^c}$.

A term t over $\mathcal{V}$ is a constant or a variable; different constants represent different values (unique name assumption). In practice, when each decision maker gives her vocabulary she gives the set of alternatives, the set of criteria and the set of relations. Then, as we will see later, each decision maker gives for each alternative its profile, with the set of profiles for all alternatives being the Profile of the decision maker corresponding to her set of alternatives and criteria.

Example 1. *For instance, for a decision maker $I \in C_{\mathcal{I}}$, we consider three alternatives $\mathcal{C}_{A,\mathcal{I}} = \{A_1, A_2, A_3\}$ and the criteria $\mathcal{P}_{C,\mathcal{I}} = \{c_1, c_2\}$ where c_1 is a binary predicate and c_2 is a ternary predicate. Please note that both predicates represent an alternative who has some values - either by means of a binary predicate or by the means of a ternary (or more). We do not represent here, by the means of $\mathcal{P}_{C,\mathcal{I}}$ that a criterion is preferred to the other or two values are preferred. This is done outside the FOL formalism.*

$\mathcal{C}_V = \{V_1, V_2, V_3, V_4, V_5\}$ *and more precisely:*

- $\mathcal{C}_V^{c_1} = \{V_1, V_2\}$ *and* $\mathcal{C}_V^{c_2} = \{V_3, V_4, V_5\}$*. This signifies that the c_1 predicate can only take $\{V_1, V_2\}$ as values and the c_2 predicate can only take $\{V_3, V_4, V_5\}$ as values.*
- $V_1 \gg_{\mathcal{C}_V^{c_1}} V_2$ *and* $V_3 \gg_{\mathcal{C}_V^{c_2}} V_4 \gg_{\mathcal{C}_V^{c_2}} V_5$

We also consider $\mathcal{P}_R = \{p_1, p_2, p_3\}$ with p_1 and p_2 unary predicate and p_3 binary and $C_C = \{C_1, C_2, C_3\}$.

An **atomic formula** (or atom) over $\mathcal{V}$ is of the form $p(t_1, ..., t_n)$ where $p \in \mathcal{P}$ is an n-ary predicate, and $t_1, ..., t_n$ are terms. A ground atom is an atom with no variables. A conjunction of atoms is called a *conjunct*. A conjunction of ground atoms is called a *ground conjunct*. A variable in an atom is free if it is not in the scope of any quantifier. A formula is *closed* if it has no free variables. A closed formula is called a *sentence*. Factual knowledge (a fact) about the world is represented by *ground atoms*. In the Existential Rules framework this concept has been extended so that a fact on $\mathcal{V}$ is the existential closure of a conjunction of atoms over $\mathcal{V}$ [15]. Let F be a fact, we denote by $terms(F)$ (resp. $vars(F)$) the set of terms (resp. variables) that occur in F.

In the following, we consider a special kind of fact called **profile**. This will allow us to give, for each alternative, the set of criteria we consider for the decision process along to their corresponding values. A *profile* for alternative $A \in \mathcal{C}_A$ and for decision maker $I \in \mathcal{C}_{\mathcal{I}}$, denoted $profile_I(A)$, is a conjunction of facts of the form $p_0(A) \wedge \bigwedge_{1 \leq i \leq n} p_i(A, \mathbf{t})$ with A representing an alternative (i.e., $A \in \mathcal{C}_A$), p_0 an unary predicate in $\mathcal{P}_R$ representing the type of the alternative, $\{p_i | 1 \leq i \leq n\}$ a set of criteria that apply for the alternative A and $\mathbf{t}$ a vector of the other terms except A of the respective criterion predicate. Note that, in a decision problem multiple types of alternatives can be proposed as the solution and hence, the need to define p_0.

Example 1 (cont.)**.** *For the three alternatives $\mathcal{C}_{A,\mathcal{I}} = \{A_1, A_2, A_3\}$ and the predicates and constants defined in the previous examples we have three profiles which are the following and correspond to each of the three alternatives:*

- $profile_I(A_1) = p_1(A_1) \wedge c_1(A_1, V_1) \wedge c_2(A_1, V_3, V_4)$
- $profile_I(A_2) = p_1(A_2) \wedge c_1(A_2, V_1) \wedge c_2(A_2, V_4, V_5)$
- $profile_I(A_3) = p_1(A_3) \wedge c_1(A_3, V_2) \wedge c_2(A_3, V_3, V_5)$

Each decision maker $I \in \mathcal{C}_\mathcal{I}$ will express her set of profiles over the set of alternatives $\mathcal{C}_{A,\mathcal{I}}$. This is denoted by $Profile_I(\mathcal{C}_{A,\mathcal{I}})$. Hence, $Profile_I(\mathcal{C}_{A,\mathcal{I}}) = \bigwedge_{A_k \in \mathcal{C}_{A,\mathcal{I}}} profile_I(A_k)$. When I is implicit we denote it simply by $Profile(\mathcal{C}_A) = \bigwedge_{A_k \in \mathcal{C}_{A,\mathcal{I}}} profile_I(A_k)$.

Example 1 (cont.)**.** *If we consider that the profiles in the previous example were given by one decision maker then, for* $\mathcal{C}_A = \{A_1, A_2, A_3\}$ *then* $Profile(\mathcal{C}_A) = p_1(A_1) \wedge c_1(A_1, V_1) \wedge c_2(A_1, V_3, V_4) \wedge p_1(A_2) \wedge c_1(A_2, V_1) \wedge c_2(A_2, V_4, V_5) \wedge p_1(A_3) \wedge c_1(A_3, V_2) \wedge c_2(A_3, V_3, V_5)$*.*

We recall the notions of *substitution* and *homomorphism* between facts to be used in order to reason about profiles or facts in general. Given a set of variables $\mathcal{X}$ and a set of terms $\mathcal{T}$, a **substitution** σ of $\mathcal{X}$ by $\mathcal{T}$ (notation $\sigma : \mathcal{X} \rightarrow \mathcal{T}$) is a function from $\mathcal{X}$ to $\mathcal{T}$. Given a fact F, $\sigma(F)$ denotes the fact obtained from F by replacing each occurrence of $x \in X \cap vars(F)$ by $\sigma(x)$. A **homomorphism** from a fact F to a fact F' is a substitution σ of $vars(F)$ by (a subset of) $terms(F')$ such that $\sigma(F) \subseteq F'$.

A conjunctive query (CQ) has the following form: $\mathcal{Q} = \texttt{ans}(x_1, \ldots, x_k) \leftarrow B$, where B (the "body" of $\mathcal{Q}$) is an existential closed atom or a conjunction of existential closed atoms, and $x_1, \ldots, x_k$ are variables that occur in B and **ans** is a special k-ary predicate, whose elements are used to build an answer. Given a set of facts $\mathcal{F}$, an answer to $\mathcal{Q}$ in $\mathcal{F}$ is a tuple of constants $(D_1, \ldots, D_k)$ such that there is a homomorphism σ from B to $\mathcal{F}$, with $\sigma(ans(x_1, x_2, \ldots x_k)) = \sigma(ans(D_1, D_2, \ldots D_k))$. If $k = 0$, i.e., $\mathcal{Q} = \texttt{ans}() \leftarrow B$, $\mathcal{Q}$ is called a Boolean conjunctive query, the unique answer to $\mathcal{Q}$ is the empty tuple if there is a homomorphism from B to $\mathcal{F}$, otherwise there is no answer to $\mathcal{Q}$. Note that a query $\mathcal{Q}$ can be shortly referred to by its body B. For its simplicity, this notation will be used hereafter.

Example 1 (cont.)**.** *For example, if we want to retrieve the alternatives in the profiles that have the value* V_1 *for the criterion* c_1 *then we can write the following query:* $\mathcal{Q} = \texttt{ans}(x_1) \leftarrow p_1(x_1) \wedge c_1(x_1, V_1)$*. The set of possible answers to the query in* $Profile(\mathcal{C}_A)$ *will be* $\{(A_1), (A_2)\}$*.*

In order to represent enriched knowledge about the facts, we use rules that encode domain-specific knowledge. Rules are regarded as an ontological layer that reinforces the expressiveness of the knowledge base. Rules are logical formulae that allow us to infer new facts (conclusion) from existing facts (hypothesis). **Existential rules** [15, 35] introduce new variables in the conclusion having ability to represent unknown individuals (also known in database community as *value invention* [1]). This form of rules is also known as tuple-generating dependencies in database community [51]. We denote by $\mathbf{x}$ in a bold font a vector of variables. An *existential rule* (or simply a rule) is a closed formula of the form $R = \forall \mathbf{x} \forall \mathbf{y}(B \rightarrow \exists \mathbf{z} H)$, where B and H are conjuncts, with $vars(B) = \mathbf{x} \cup \mathbf{y}$, and $vars(H) = \mathbf{x} \cup \mathbf{z}$. The variables in vector $\mathbf{z}$ are called the existential variables of the rule R. B and H are respectively called the *body* and the *head* of R. We denote them respectively $body(R)$ for B and $head(R)$ for H.

Example 1 (cont.)**.** *Examples of rules in the example considered before could include:*

- $R_1 = \forall x(c_1(x, V_1) \rightarrow p_1(x))$
- $R_2 = \forall x(c_1(x, V_2) \rightarrow p_2(x))$
- $R_3 = \forall x(c_2(x, V_3, V_4) \rightarrow p_2(x))$
- $R_4 = \forall x \forall y(c_2(x, y, V_4) \rightarrow \exists z p_3(x, z))$

Real examples of the above rules could be that an alternative that has value V_1 for price is a cheap alternative (R_1) and an alternative that has value V_2 for price is an expensive alternative (R_2). R_3 might state that an alternative that has good reviews in the V_3 journal is an expensive alternative. The last rule might say that an alternative that has good reviews (i.e., value V_4) in all journals (i.e. variable y) will be discounted in at least one shop (i.e., the existential variable z).

Existential rules are more expressive than Description Logics as they can represent complex relations between individuals and overcome the "cycle on variables" [38]. Another important aspect of the existential rules framework is the possibility of having *unbounded predicate arity*, i.e, predicates with an arbitrary number of parameters. Entailment is undecidable for general existential rules [18]. However, many classes of existential rules that ensure decidability (while keeping expressiveness) have been studied (see [15]). In this paper and for practical reasons we work on such classes.

We also account for a special kind of rules called *negative constraints*, i.e., knowledge that imposes constraints about the world, also known as denial constraints in databases [35]. A negative constraint (or simply a constraint) is a rule of the form $N = \forall \mathbf{x}(B \rightarrow \bot)$, where $vars(B) = \mathbf{x}$. Negative constraints in the existential rules framework fully capture *concept disjointness* of DLs. From now on we omit quantifiers in front of formulae as there is no ambiguity.

Example 1 (cont.)**.** *An example of negative constraint on the vocabulary of the example considered before could be $\forall x(p_1(x) \land p_2(x) \rightarrow \bot)$ (for example an alternative cannot be expensive and cheap at the same time).*

An individual decision maker's I **knowledge base** is a tuple $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})_I$ (denoted as $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ when there is no ambiguity) of finite sets of facts (thus including profiles), rules and negative constraints respectively. Reasoning with a knowledge base of facts, rules and negative constraints is done via a mechanism called saturation of facts by the rules. In order to define saturation we need to define rule applicability on facts.

A rule $R = \forall \mathbf{x} \forall \mathbf{y}(B \rightarrow \exists \mathbf{z} H)$ is **applicable** [15] to a fact F if there exists a homomorphism σ from B to F. The *application of* R *to* F *w.r.t.* σ produces a fact $\alpha(F, R, \sigma) = F \cup \sigma(safe(H))$, where $\mathtt{safe}(H)$ is obtained from H by replacing existential variables with fresh variables (not used variables). $\alpha(F, R, \sigma)$ is said to be an **immediate derivation** from F. Let F be a fact and $\mathcal{R}$ be a set of rules. A fact F' is called an **$\mathcal{R}$-derivation** of F if there is a finite sequence (called the **derivation sequence**) $\langle F_0 = F, ..., F_n = F' \rangle$ such that for all $0 \leq i < n$ there is a rule $R \subseteq \mathcal{R}$ which is applicable to F_i and F_{i+1} is an immediate derivation from F_i. The saturation operator ($C\ell$) can be seen as a *fixed-point operator* where we denote by $C\ell^*_{\mathcal{R}}(\mathcal{F})$ the saturation of $\mathcal{F}$ with respect to $\mathcal{R}$. Note that $C\ell^*_{\mathcal{R}}(\mathcal{F})$ is a finite set [16] for the classes of existential rules considered in this paper.

Example 1 (cont.)**.** *Let us consider again the profiles for* $\mathcal{C}_A = \{A_1, A_2, A_3\}$*:* $Profile(\mathcal{C}_A) = p_1(A_1) \wedge c_1(A_1, V_1) \wedge c_2(A_1, V_3, V_4) \wedge p_1(A_2) \wedge c_1(A_2, V_1) \wedge c_2(A_2, V_4, V_5) \wedge p_1(A_3) \wedge c_1(A_3, V_2) \wedge c_2(A_3, V_3, V_5)$.

R_1 *and* R_2 *are applicable on* $Profile(\mathcal{C}_A)$ *yielding the new facts* $p_1(A_1)$, $p_1(A_2)$ *and* $p_2(A_3)$. R_3 *is also applicable on* $Profile(\mathcal{C}_A)$ *yielding the new fact* $p_2(A_1)$*. The facts* $p_1(A_1)$ *and* $p_2(A_1)$ *will trigger the negative constraint* $\forall x(p_1(x) \wedge p_2(x) \rightarrow \bot)$ *(thus, as we will see in the next subsection) rendering* $Profile(\mathcal{C}_A)$ *inconsistent.*

In the next subsection we will see how to reason with facts in the presence of inconsistency using repair based methods.

3.3 Multi-criteria Decision Making as Repair Techniques

In what follows we recall the formal definition of *inconsistency* in the existential rules framework; then we introduce the subset-repairing techniques which is inspired by the work from the database community [39] and Description Logics [69, 23]. Please note that the aim of this paper is to show how the decision problem can be modelled as an inconsistent knowledge base. For this reason we will simply favor the modelling part explaining the various choices made and will not go deeper into the inconsistency tolerant semantics aspect.

Definition 12 (Inconsistency)**.** *A set of facts* $\mathcal{F}$ *is inconsistent with respect to a set of rules* $\mathcal{R}$ *and negative constraints* $\mathcal{N}$ *(or inconsistent for short) if and only if there exists a constraint* $N \in \mathcal{N}$ *such that* $C\ell^*_{\mathcal{R}}(\mathcal{F}) \models body(N)$.

This means that the set of facts *violates* the negative constraint N or triggers it. Correspondingly, a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ is inconsistent (with respect to $\mathcal{R}$ and $\mathcal{N}$) if and only if there exists a set of facts $\mathcal{F}' \subseteq \mathcal{F}$ such that $\mathcal{F}'$ is inconsistent. An alternative writing is $C\ell^*_{\mathcal{R}}(\mathcal{F}) \models \bot$.

One way to cope with inconsistency is to construct *maximal consistent subsets* of the knowledge base [77]. This corresponds to "Data Repairs" [8]. A data repair of a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ is a set of facts $\mathcal{F}'$ such that $\mathcal{F}'$ is consistent and there exists no consistent subset of $\mathcal{F}$ that strictly contains $\mathcal{F}'$ [69].

Since repairs are computed exclusively on the set of facts and given that the factual part of the knowledge base is the only source of inconsistency we, from now on, abuse slightly the notation and refer to $\mathcal{K}'$ by its set of facts $\mathcal{F}'$. The set of all repairs of $\mathcal{K}$ is denoted by $\mathcal{R}epair(\mathcal{K})$.

For one individual decision maker $I \in \mathcal{C}_{\mathcal{I}}$ and the set of alternatives $\mathcal{C}_A$ we use repair techniques in order to obtain one single, maximal consistent subset of $Profile(\mathcal{C}_A)$.

For a given set of alternatives $\mathcal{C}_A$ and for each of the individual decision makers $I \in \mathcal{C}_{\mathcal{I}}$, the $Profile_I(\mathcal{C}_A)$ is consistent wrt to the commonly agreed set of rules and negative constraints [1]. Note that we need to have a common structure for all the profiles and we do not consider that the preferences of an individual decision maker are inconsistent.

The first step is to construct $Profile_I(\mathcal{C}_A)$ for every decision maker $I \in \mathcal{C}_{\mathcal{I}}$.

Example 2. *In the following example, we present a multi-criteria decision problem that will guide us throughout the modeling section. The multi-criteria decision problem illustrated is a complicated problem every young parent faces: what pushchair to buy (preferably before the baby arrives)? We have the set of alternatives* $\mathcal{C}_A = \{Yoyo, Chicco, Inglesina\}$*, the set of decision makers* $\mathcal{C}_{\mathcal{I}} = \{M, P, J\}$*, where* $I \in \mathcal{C}_{\mathcal{I}}$ *are constants, and the set of criteria* $\mathcal{P}_c = \{price, weight, transportable\}$*. Note that the criterion transportable is a ternary predicate*$(a, size, foldable)$*. The preferences of the decision maker* M *for all the alternatives are included in the* $Profile_M(\mathcal{C}_A) = profile_M(Yoyo) \wedge profile_M(Chicco) \wedge profile_M(Inglesina)$ *, where:*

- $profile_M(Yoyo) = pushchair(Yoyo) \wedge price(Yoyo, 500) \wedge weight(Yoyo, 3)$ $\wedge\, transportable(Yoyo, large, yes)$
- $profile_M(Chicco) = pushchair(Chicco) \wedge price(Chicco, 300) \wedge$ $weight(Chicco, 4) \wedge transportable(Chicco, large, no)$
- $profile_M(Inglesina) = pushchair(Inglesina) \wedge price(Inglesina, 400)$ $\wedge\, weight(Inglesina, 2) \wedge transportable(Inglesina, medium, yes)$

[1] Please remember that we are in the OBDA case where the inconsistency can only come from facts thus we agree upon a common ontology (i.e., set of rules and negative constraints) a priori.

At this step it is very important to mention the fact that each decision maker also has a (strict) total preference ordering on the set of criteria they consider in their profiles.

Inconsistencies will also arise at this step are therefore due to several factors:

- The set of criteria is not the same for all decision makers.
- The set of alternatives is not the same for all decision makers.
- The preferences of criteria are not the same for all decision makers.
- The value of an alternative for a given criterion is not the same for all decision makers.
- The subjective preferences (i.e., the evaluation ratings) given by the decision makers for an alternative on a given criterion are not the same.

At this step we consider a fresh set of constants that correspond to those criteria that the decision maker I considers in her aggregation function. We also need to consider the set of constants naming the decision makers, i.e., for every $c \in \mathcal{P}(c)$ the new constant name is C^*. We introduce two meta predicates: *consider* (a ternary predicate taking a decision maker's identifier, an alternative and a constant corresponding to the criterium name) and *preferred* (a ternary predicate stating that I prefers the criterion i to criterion j).

3.3.1 Inconsistency regarding the consideration of different criteria

Each decision maker $I \in \mathcal{C}_\mathcal{I}$ gives a set of rules of the form $\mathcal{R}_I = \{\forall A \in \mathcal{C}_A \; type(A) \wedge \mathcal{J}_{I_i} \rightarrow consider(I, A, C^*)\}$, where C^* is the constant that corresponds to a criterion that I considers for alternative A. We have that *type* is a predicate and A is in its interpretation. By $\mathcal{J}_i$, we denote the justification made by the decision maker for her rule R_i. Negative constraints occur when different decision makers do not consider the same criteria. Therefore negative constraints are of the form $\forall I, J \in \mathcal{C}_\mathcal{I} \forall A \in \mathcal{C}_A \forall C^* \in \mathcal{C}_C, consider(I, A, C^*) \wedge not\_consider(J, A, C^*) \wedge type(A) \wedge diff(I, J) \rightarrow \perp$. The predicate $not\_consider(I, A, C^*)$ corresponds to the case where criterion C^* is not considered by I for her decision about alternative A and predicate $diff(I, J)$ means that we have two different decision makers I and J.

Let us now illustrate the modelling choices above using the ongoing example.

Example 3. *Given the set of decision makers $\mathcal{C}_\mathcal{I} = \{M, P\}$, let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base with:*

- *Set of facts* $\mathcal{F} = \{\mathcal{J}_{M1}, \mathcal{J}_{M2}, \mathcal{J}_{P1}, \mathcal{J}_{P2}, pushchair(Yoyo), pushchair(Chicco)\}$

- *Set of rules* $\mathcal{R} = \mathcal{R}_M \cup \mathcal{R}_P$

$$\begin{aligned}\mathcal{R}_P =& \{\forall a, pushchair(a) \wedge \mathcal{J}_{P1} \rightarrow consider(P, a, price^*),\\ & \forall a, pushchair(a) \wedge \mathcal{J}_{P2} \rightarrow consider(P, a, weight^*)\}\\ \mathcal{R}_M =& \{\forall a, pushchair(a) \wedge \mathcal{J}_{M1} \rightarrow consider(M, a, price^*),\\ & \forall a, pushchair(a) \wedge \mathcal{J}_{M2} \rightarrow not\_consider(M, a, weight^*)\}\end{aligned}$$

- *Set of negative constraints* $\mathcal{N}$ *with*

$$\begin{aligned}\mathcal{N} =& \{\forall i \forall a \forall c_k, consider(i, a, c_k) \wedge not\_consider(j, a, c_k^*) \wedge pushchair(a)\\ & \wedge diff(i, j) \rightarrow \perp\}\end{aligned}$$

By applying the rules we obtain the set

$$\begin{aligned}\mathcal{F}^* = \mathcal{F} \cup \{& consider(M, Yoyo, price^*),\\ & consider(P, Yoyo, price^*),\\ & consider(P, Yoyo, weight^*),\\ & not\_consider(M, Yoyo, weight^*),\\ & consider(M, Chicco, price^*),\\ & consider(P, Chicco, price^*),\\ & consider(P, Chicco, weight^*),\\ & not\_consider(M, Chicco, weight^*)\}\end{aligned}$$

We can see that the negative constraint is triggered twice. Firstly, P considers the weight for pushchair $Yoyo$ whereas M does not. Secondly, P also considers the weight for pushchair $Chico$ whereas M does not.

3.3.2 Inconsistency regarding the consideration of different alternatives

Each decision maker $I \in \mathcal{C}_\mathcal{I}$ gives a set of rules of the form $R_I = \{\forall A \in \mathcal{C}_A \; type(A) \wedge \mathcal{J}_{I_i} \rightarrow alternative(A, I)\}$ which corresponds to the alternatives considered by I. Inconsistencies occur when different decision makers do not consider the same alternatives. Therefore negative constraints are of the form $\forall I, J \in \mathcal{C}_\mathcal{I} \forall A \in \mathcal{C}_A \; alternative(A, I) \wedge not\_alternative(A, J) \wedge diff(I, J) \rightarrow \perp$. The predicate $not\_alternative(A, I)$ corresponds to the case where alternative A is not considered by I for her decision and predicate $diff(I, J)$ means that we have two different decision makers I and J.

Example 4. *Given the set of decision makers $\mathcal{C}_\mathcal{I} = \{M, P\}$, let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base with:*

- *Set of facts* $\mathcal{F} = \{\mathcal{J}_M, \mathcal{J}_P, pushchair(Yoyo), pushchair(Chicco)\}$

- *Set of rules* $\mathcal{R} = \mathcal{R}_P \cup \mathcal{R}_M$

$$\begin{aligned}\mathcal{R}_P =& \{\forall a, pushchair(a) \wedge \mathcal{J}_P \rightarrow alternative(Yoyo, P)\\ & \wedge not\_alternative(Chicco, P)\}\\ \mathcal{R}_M =& \{\forall a, pushchair(a) \wedge \mathcal{J}_M \rightarrow alternative(Yoyo, M)\\ & \wedge alternative(Chicco, M)\}\end{aligned}$$

- *Set of negative constraints* $\mathcal{N}$ *with*

$$\mathcal{N} = \{\forall j \forall i \forall a\ alternative(a, i) \wedge not\_alternative(a, j) \wedge diff(i, j) \rightarrow \perp\}$$

By applying the rules we obtain the set

$$\begin{aligned}\mathcal{F}^* = \mathcal{F} \cup \{& alternative(Yoyo, M),\\ & alternative(Yoyo, P),\\ & alternative(Chicco, M),\\ & not\_alternative(Chicco, P)\}\end{aligned}$$

We can see that M *is considering alternative Chicco and* P *is not considering Chicco, so the negative constraint is triggered leading to an inconsistency.*

3.3.3 Inconsistency regarding the consideration of different preferences on criteria

Each decision maker $I \in \mathcal{C}_\mathcal{I}$ gives a set of rules of the form $\mathcal{R}_I = \{\forall A \in \mathcal{C}_A, \forall C_i^*, C_j^* \in \mathcal{C}_C\ consider(I, A, C_i^*) \wedge consider(I, A, C_j^*) \wedge \mathcal{J}_{I_i} \rightarrow preferred(I, C_i^*, C_j^*)\}$ stating her preferences between criteria c_i and c_j. Inconsistencies occur when different decision makers consider different preferences over the criteria. Therefore negative constraints are of the form $\forall A \in \mathcal{C}_A \forall I, J \in \mathcal{C}_\mathcal{I} \forall C_i^*, C_j^* \in \mathcal{C}_C, preferred(I, A, C_i^*, C_j^*) \wedge not\_preferred(J, A, C_i^*, C_j^*) \wedge diff(I, J), preferred(I, A, C_i^*, C_j^*) \wedge equivalent(J, A, C_i^*, C_j^*) \wedge diff(I, J), equivalent(I, A, C_i^*, C_j^*) \wedge not\_preferred(J, A, C_i^*, C_j^*) \wedge diff(I, J) \rightarrow \perp$. The atom $not\_preferred(I, A, C_i^*, C_j^*)$ corresponds to the case where criterion C_i^* is not preferred in a pairwise comparison with criterion C_j^* by I for her decision on alternative A and predicate $equivalent(I, A, C_i^*, C_j^*)$ means that criteria C_i^* and C_j^* are equally preferred by I.

Example 5. *Given the set of decision makers* $\mathcal{C}_\mathcal{I} = \{M, P\}$*, let* $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ *be a knowledge base with:*

- *Set of facts* $\mathcal{F}$ *with*

$$\begin{aligned}\mathcal{F} =& \{\mathcal{J}_{M1}, \mathcal{J}_{M2}, \mathcal{J}_{M3}, \mathcal{J}_{P1}, \mathcal{J}_{P2}, \mathcal{J}_{P3}, pushchair(Yoyo),\\ & pushchair(Chicco)\}\end{aligned}$$

- *Set of rules* $\mathcal{R} = \mathcal{R}_P \cup \mathcal{R}_M$

$$\begin{aligned}\mathcal{R}_P =\{&\forall a, pushchair(a) \wedge \mathcal{J}_{P1} \rightarrow consider(P, a, price^*),\\ &\forall a, pushchair(a) \wedge \mathcal{J}_{P2} \rightarrow consider(P, a, weight^*),\\ &\forall a, consider(P, a, price^*) \wedge consider(P, a, weight^*) \wedge \mathcal{J}_{P3} \rightarrow\\ &not\_preferred(P, a, weight^*, price^*)\}\end{aligned}$$

$$\begin{aligned}\mathcal{R}_M =\{&\forall a, pushchair(a) \wedge \mathcal{J}_{M1} \rightarrow consider(M, a, price^*),\\ &\forall a, pushchair(a) \wedge \mathcal{J}_{M2} \rightarrow consider(M, a, weight^*),\\ &\forall a, consider(M, a, price^*) \wedge consider(M, a, weight^*) \wedge \mathcal{J}_{M3} \rightarrow\\ &preferred(M, a, weight^*, price^*)\}\end{aligned}$$

- *Set of negative constraints* $\mathcal{N}$ *with*

$$\begin{aligned}\mathcal{N} =\{&\forall a \forall i \forall j \forall c_1 \forall c_2, preferred(i, a, c_1, c_2) \wedge not\_preferred(j, a, c_1, c_2) \wedge\\ &diff(i, j) \rightarrow \bot\}\end{aligned}$$

By applying the rules we obtain the set

$$\begin{aligned}\mathcal{F}^* = \mathcal{F} \cup \{&consider(M, Yoyo, price^*),\\ &consider(P, Yoyo, price^*),\\ &consider(P, Yoyo, weight^*),\\ &consider(M, Yoyo, weight^*),\\ &preferred(M, Yoyo, weight^*, price^*),\\ &not\_preferred(P, Yoyo, weight^*, price^*)\\ &consider(M, Chicco, price^*),\\ &consider(P, Chicco, price^*),\\ &consider(P, Chicco, weight^*),\\ &consider(M, Chicco, weight^*),\\ &preferred(M, Chicco, weight^*, price^*),\\ &not\_preferred(P, Chicco, weight^*, price^*)\}\end{aligned}$$

We can see that M *and* P *have different preferences over the criteria (*$price^*$ *and* $weight^*$*) for each of the alternatives, i.e.,* M *considers criterion weight more important than price while* P *considers the opposite. Hence, the negative constraint is triggered leading to an inconsistency.*

3.3.4 Inconsistency on the criteria values of the decision makers over the alternatives

Each decision maker $I \in \mathcal{C}_\mathcal{I}$ gives a set of rules of the form $\mathcal{R}_I = \{\forall A \in \mathcal{C}_A\ type(A) \wedge \mathcal{J}_{I_i} \rightarrow has\_value(I, A, C^*, V)\}$ that corresponds to the value $V \in \mathcal{C}_V^c$ I gives for criterion C^*

for alternative A. Inconsistencies occur when different decision makers have different valuation for the same alternative on a specific criterion. The value corresponds to a metric that is related to real data. Although one could think that, since value is objective, it would be the same for all the decision makers, this is not the case in real life simply because not all the decision makers have the same information. Therefore, negative constraints are of the form $\forall I, J \in \mathcal{C}_{\mathcal{I}} \forall A \in \mathcal{C}_A \forall C^* \in \mathcal{C}_C \forall V_1, V_2 \in \mathcal{C}_V^c, has\_value(I, A, C^*, V_1) \wedge has\_value(J, A, C^*, V_2) \wedge type(A) \wedge diff(I, J) \wedge diff(V_1, V_2) \rightarrow \perp$.

Example 6. *Given the set of decision makers $\mathcal{C}_{\mathcal{I}} = \{M, P\}$, let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base with:*

- *Set of facts $\mathcal{F} = \{\mathcal{J}_{M1}, \mathcal{J}_{M2}, \mathcal{J}_{P1}, \mathcal{J}_{P2}, pushchair(Yoyo), pushchair(Chicco)\}$*
- *Set of rules $\mathcal{R} = \mathcal{R}_P \cup \mathcal{R}_M$*

$$\begin{aligned}\mathcal{R}_P =& \{\forall a, pushchair(a) \wedge \mathcal{J}_{P1} \rightarrow has\_value(P, Yoyo, price^*, 90),\\ & \forall a, pushchair(a) \wedge \mathcal{J}_{P2} \rightarrow has\_value(P, Chicco, price^*, 80),\\ \mathcal{R}_M =& \{\forall a, pushchair(a) \wedge \mathcal{J}_{M1} \rightarrow has\_value(M, Yoyo, price^*, 70),\\ & \forall a, pushchair(a) \wedge \mathcal{J}_{M2} \rightarrow has\_value(M, Chicco, price^*, 60)\}\end{aligned}$$

- *Set of negative constraints $\mathcal{N} = \{\forall I, J \in \mathcal{C}_{\mathcal{I}} \forall A \in \mathcal{C}_A \forall C^* \in \mathcal{C}_C \forall V_1, V_2 \in \mathcal{C}_V^c, has\_value(I, A, C^*, V_1) \wedge has\_value(J, A, C^*, V_2) \wedge type(A) \wedge diff(I, J) \wedge diff(V_1, V_2) \rightarrow \perp\}$*

By applying the rules we obtain the set

$$\begin{aligned}\mathcal{F}^* = \mathcal{F} \cup \{& has\_value(M, Yoyo, price^*, 70),\\ & has\_value(P, Yoyo, price^*, 90)\\ & has\_value(M, Chicco, price^*, 60),\\ & has\_value(P, Chicco, price^*, 80\}\end{aligned}$$

We can see that M and P have different values in price criterion for both alternatives (70 and 90 for Yoyo, 60 and 80 for Chicco), so the negative constraint is triggered leading to an inconsistency. Even if the value for price can be objective, inconsistency can occur due to lack of information or different source of information, e.g., different stores can have different prices on the same product.

3.3.5 Inconsistency on the criteria evaluation ratings (preferences) of the decision makers over the alternatives

Each decision maker $I \in \mathcal{C}_{\mathcal{I}}$ gives a set of rules of the form $\mathcal{R}_I = \{\forall A \in \mathcal{C}_A\ type(A) \wedge \mathcal{J}_{I_i} \rightarrow has\_rating(I, A, C^*, R)$ that corresponds to the evaluation rating $R \in \mathcal{C}_R^c$ that I gives for criterion $C^* \in \mathcal{C}_C$ for alternative A. Inconsistencies occur when different decision makers have different preferences, i.e., evaluation ratings for the same alternative on a specific criterion. The evaluation rating corresponds to a metric that is related to the preferences of the decision makers. The preferences of the decision makers are subjective and this can be seen through the inconsistencies of the evaluation ratings. Therefore, negative constraints are of the form $\forall I, J \in \mathcal{C}_\mathcal{I} \forall A \in \mathcal{C}_A \forall C^* \in \mathcal{C}_C \forall R_1, R_2 \in \mathcal{C}_R^c, has\_rating(I, A, C^*, R_1) \wedge has\_rating(J, A, C^*, R_2) \wedge type(A) \wedge diff(I, J) \wedge diff(R_1, R_2) \rightarrow \perp$..

Example 7. *In the following example "vfm" stands for the "value for money" criterion. Given the set of decision makers $\mathcal{C}_\mathcal{I} = \{M, P\}$, let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base with:*

- *Set of facts* $\mathcal{F} = \{\mathcal{J}_{M1}, \mathcal{J}_{M2}, \mathcal{J}_{P1}, \mathcal{J}_{P2}, pushchair(Yoyo), pushchair(Chicco)\}$
- *Set of rules* $\mathcal{R} = \mathcal{R}_P \cup \mathcal{R}_M$

$$\begin{aligned}\mathcal{R}_P =& \{\forall a, pushchair(a) \wedge \mathcal{J}_{P1} \rightarrow has\_rating(P, Yoyo, vfm^*, 9),\\ &\forall a, pushchair(a) \wedge \mathcal{J}_{P2} \rightarrow has\_rating(P, Chicco, vfm^*, 8),\\ \mathcal{R}_M =& \{\forall a, pushchair(a) \wedge \mathcal{J}_{M1} \rightarrow has\_rating(M, Yoyo, vfm^*, 7),\\ &\forall a, pushchair(a) \wedge \mathcal{J}_{M2} \rightarrow has\_rating(M, Chicco, vfm^*, 7)\}\end{aligned}$$

- *Set of negative constraints* $\mathcal{N}$ *with*

$$\begin{aligned}\mathcal{N} =& \{\forall i \forall a \forall c_k \forall v_n \forall v_{n\prime}, n \in [1 \dots N], has\_rating(i, a, c_k, v_n)\\ &\wedge has\_rating(j, a, c_k, v_{n\prime}) \wedge pushchair(a) \wedge diff(i, j) \wedge\\ &\exists n, n\prime \mid diff(v_n, v_{n\prime}) \rightarrow \perp\}\end{aligned}$$

By applying the rules we obtain the set

$$\begin{aligned}\mathcal{F}^* = \mathcal{F} \cup \{&has\_rating(M, Yoyo, vfm^*, 7),\\ &has\_rating(P, Yoyo, vfm^*, 9)\\ &has\_rating(M, Chicco, vfm^*, 7),\\ &has\_rating(P, Chicco, vfm^*, 8\}\end{aligned}$$

The obtained facts show the ratings given by P and M for the vfm of $Yoyo$ and $Chicco$. We can see that they have different perspective on judging the "vfm" criterion for both alternatives (M gives 7 and P gives 9 for Yoyo, while they give 7 and 8 for Chicco), so the negative constraint is triggered leading to an inconsistency.

3.4 Discussion and related work

In this work, we proposed a framework to model the multi-criteria decision problem as reasoning in presence of inconsistency problem. We used existential rules for the modelling language of the knowledge base and employed repair techniques for reasoning.

Please note that existing work considers alternative inconsistency proof semantics. In knowledge representation and reasoning argumentation is also a well known method for handling inconsistency. Several researchers have proposed the use of argumentation in decision-making. The work of Fox and Parsons [56] is among the first argumentative approaches to decision-making by clarifying the difference between argumentation for actions and argumentation for beliefs. Most of the argumentative approaches ([60], [72]) aim to select the best option but certain approaches (such as those by Amgoud and Prade [7]) use a two step process that allows to further analyse the different options presented to the user. Using argumentation techniques for reasoning with existential rules knowledge bases was proven to yield semantically equivalent results as to repair based techniques. Furthermore, argumentation has been long investigated for its principled human computer interaction advantages (see for instance the results of using argumentation for handling inconsistency in existential rules of [9, 11, 10]). However, in this context, argumentation is computationally extensive process that involves a large overhead due to the computation of all arguments and attacks over a knowledge base.

Chapter 4

Implementation of methods and algorithms of Social Choice Theory and their subsequent use in real applications

The following chapter is composed of two main sections, both of which refer to practical applications and software implementation of methods regarding social choice and collective decision-making. In the first section we present a decision support tool which is based on computational social choice and argumentation and is being applied in problems of agricultural engineering. We describe extensively the general architecture of system, as well as the implementation details and its application on a real problem. The abovementioned work has been published to the International Journal of Agricultural and Environmental Information Systems [61]. The second section of this chapter is dedicated to the presentation of the algorithms that implement method from social choice and argumentation and were used to support the theoretical results of this research. These algorithms can be found at the core of future work regarding software tools of reasoning based methods on collective decision making.

4.1 A Decision Support Tool for Agricultural Applications Based on Computational Social Choice and Argumentation

Abstract

In the current section, we describe an applied procedure to support collective decision making for applications in agriculture. The problem we are facing in this paper is how to reach the best decision regarding issues coming from agricultural engineering with the aid of Computational Social Choice and Argumentation Framework. In the literature of decision-making, several approaches from the domains of Computational Social Choice and Argumentation Framework have been used autonomously to support

decisions. It is our belief that with the combination of these two fields the authors can propose socially fair decisions which take into account both (1) the involved agents' preferences and (2) the justifications behind these preferences. Therefore, in this section we present the implementation of a software tool for decision-making which is composed of two main systems, i.e., the social choice system and the deliberation system. In this article, the authors describe thoroughly the social choice system of our tool and how it can be applied to different alternatives on the valorization of materials coming from agriculture. As an example, that is demonstrated an application of our tool in the context of Ecobiocap European project where several decision problems are to be addressed. These decision problems consist in finding the best solutions for questions regarding food packaging and end-of-life management.

4.1.1 Introduction

Collective decision-making and preference aggregation are widely used in modern societies. The most well-known example of collective decision-making is political elections but it is not the only one since there are a number of situations where the aggregation of the individual preferences is needed to take a decision. Consider for example situations such as a group of friends choosing where to have dinner or a committee board choosing what is the best strategy for the company. In all these cases what we are seeking is a way to fairly aggregate the preferences of the individual agents into a collective preference and thus obtain a decision which satisfies the group as a total. This setting can be directly applied also in the decision-making for agricultural problems. Such an example is a problem where the decision lays in evaluating the interest and potential of marketing new generation packaging made of agro-waste materials. Here, the consumers are asked to express preferences among different packaging.

We are studying the classical collective decision-making problem, where we have a set of alternative options and a set of agents and each agent is called to express her preference over the alternatives by producing a linear order on them. In most of these collective decision-making problems the preference aggregation is done by using simple aggregation methods, such as the plurality voting rule, and the tools that are used are simple and not even intended to serve this purpose. For example, doodle is one such unsuitable tool that is used for preference aggregation while its original functionality was for scheduling joint activities. Therefore, we propose a procedure for supporting more complex collective decision-making problems, which can be directly applied in the context of agricultural applications. Our objective is to expand this classical collective decision-making problem by asking the agents to consider the reasoning behind the linear ordering of the alternatives. Hence, the goal of this work is to build, i.e., design and implement, a software tool for decision-making that takes into account theoretical insights from social choice and argumentation in order to propose social fair decisions that take into account the preferences of the agents and the reasoning behind these preferences.

Related work

The application of argumentation framework and social choice having as a goal the support of Decision Analysis is a relatively new field; however there has been significant research towards decision-making on both of these fields independently.

Multi-Criteria Decision Analysis (MCDA) has first started evolving in the 60s when Benayoun, Roy, and Sussman ([19],[78]) introduced the class of ELECTRE methods for aggregating preferences expressed on multiple criteria. The ELECTRE methods set the foundations for the "outranking methods" ([73], [79]) which was the first step of integrating notions of social choice into decision-making. MCDA and social choice are two closely correlated fields whose objective is to aggregate the partial preferences into a collective preference. Arrow and Raynaud ([12]) were the first that presented a general exploration of the links between social choice and MCDA. Voting methods from social choice theory have been integrated in the analysis of some popular aggregation methods in multi-criteria decision-aiding. Let us mention, for example, the Condorcet method, on which the ordinal methods in multi-criteria analysis, e.g., ([79],[80]) are based and the Borda method on which the cardinal ones, e.g., ([64],[83]) are based.

Several researchers have proposed the use of argumentation in decision-making analysis. The work of Fox and Parsons [56] is one of the first works that tried to deploy an argumentative approach to decision-making stating the difference between argumentation for actions and argumentation for beliefs. The objective of most of the argumentation-based approaches ([60], [72]) is to select the best solution (alternative option) compared to decision analysis, where the objective can have several different problem statements, i.e., choosing, rejecting, ranking, classifying the set of alternatives. Regarding the aggregation, several approaches ([4],[28]) used procedures based only on the number or the strength of arguments, while in decision analysis many aggregation procedures have been proposed. Another seminal work towards the usage of argumentation frameworks in decision-making is the one by ([7]) which proposes an abstract argumentation-based framework for decision-making. The model follows a 2-step procedure where at first the arguments for beliefs and options are built and at the second step we have pairwise comparisons of the options using decision principles. Also, our team adopted argumentation framework in decision-making regarding applications from the agricultural domain ([82]) and ([85]).

Recently, significant research on the applicative side of Computational Social Choice has been deployed. For instance, some web-based tools for preference aggregation that allow users to create their own polls and conduct voting have been created. The objective of these web applications is to help collective decision-making by facilitating the voting's procedure for the non-experts. Among them, the three most indicative ones are Pnyx ([32]), Robovote (Merlob, B., Procaccia, A. D., Shah, N., & Wang, P., 2016), and Whale (Bouveret, 2010). Observe that these poll tools are purely social choice oriented and are used for obtaining a collective decision in a single poll. In contrast, our tool is a decision-making tool that allows to derive decisions, based on preferences and justifications, for multiple decision problems.

Our work

Our results include the design and the implementation of a decision-making tool, that will be used for agricultural material valorization. Our goal is to design a sophisticated tool that is adapted to the needs of surveys conducted for agricultural problems and produces decisions based on the justified preferences of the agents. Therefore, we divide the tool into two main systems, i.e., the social choice system and the deliberation system. These systems can act independently or can be combined to support a decision. The social choice system relies on social choice techniques and allows for the computation of a "socially fair" global ranking of alternatives thanks to the aggregation of elicited individuals' rankings. The deliberation system will provide the argumentation framework which will allow for the computation of justifications on these individuals' rankings. It aims at allowing the "correction" of misinformed or incomplete information by using justifications coming from the different agents. Therefore, with the implementation of these two systems we adapt social choice theory and argumentation in order to have better decisions, in the sense of having justified preferences and their fair collective aggregation. In order to demonstrate the functionality of our tool, we will apply the software procedure on a decision problem extracted from an existing use-case that was conducted for the needs of the Ecobiocap project. The use-case objective is to evaluate the interest of consumers in new-generation packaging. However, note that, this use-case is not restrictive on the applicative usage of the software as we are considering a more general framework for other future possible applications and hence, the tool allows for multiple decision problems.

4.1.2 Problem Description

We are considering the problem of taking a decision regarding valorization options for agricultural materials with the aid of Computational Social Choice and Argumentation Framework. Therefore, the problem is formulated as follows. As input, we have a set of alternative options A which will be called *alternatives* and a set of agents N, that will elicit justified preferences on the alternative options. We are considering the case where the agents' preferences are expressed by justified rankings, i.e., each agent provides a total order on the alternatives and a justification for this total order. The collection of the linear orders for all the agents is called the *preference profile*. In the classical social choice theory, the aggregation of the preferences is computed by a voting rule, i.e., the preference profile is reported to a *voting rule* (method), which then singles out the winning alternative and the ranking of the remaining ones or a set of winners. The agents' preferences and the justifications are used to build the arguments and the argumentation framework AF, whose role is to help us elicit the justified preferences.

One should note that an evaluation given by the agent over a set of criteria for the alternatives can serve as justifications for the agent's total order. Taking that into consideration we can see that criteria based justification can serve as a special case of the multi-criteria decision-making problem.

4.1.3 General architecture of the decision support software

In this section we are presenting the general architecture of the decision support software. Our proposal includes the design and the implementation of a decision-making tool, which is split into two main systems, i.e., the social choice system and the deliberation system. Each one of the systems can act as a stand-alone decision tool but they can also be linked together in order to provide a decision. Our tool is composed of the following four subsystems: the data collecting, the voting, the argumentation and the decision subsystems.

Data collecting subsystem: The task of this subsystem is to yield the data needed as an input for the voting subsystem and the argumentation subsystem. The most common way for agents to reveal their preferences is through surveys. Hence, this subsystem takes as input the survey's "raw" data, i.e., the preferences of the agents in an abstract format, as well as the justifications behind these preferences. Note that we are considering different use-cases which have different forms of surveys and therefore this subsystem is survey-oriented and the method used to compute the output is adapted each time to the survey format. The output of the subsystem is structured information which contains a total order (ranking) of the alternatives for each agent with potential justifications supporting each preference.

Voting subsystem: This subsystem's function is to provide methods for electing the "best" alternatives (and/or criteria). Using the term "best", we mean the alternatives/criteria that correspond to the socially fairest outcome and that best reflect the preferences of the agents. To achieve that, we consider a voting setting where we fairly aggregate the set of agents' preferences and produce a ranking of the alternatives or a fixed-sized set of equally winning alternatives. We refer to Computational Social Choice techniques, e.g., voting rules, in order to aggregate the agents' individual preferences. Many voting rules have been proposed in the social choice literature which try to satisfy prominent and fundamental fairness criteria. However, due to the impossibility results by Arrow ([13]) and Gibbard-Satterthwaite ([58], [81]) there is no hope of finding a voting rule that can be "perfect", because due to these results some vital criteria can not be satisfied all at the same time. Hence, we propose various voting rules so that the decision maker will be able to select among them.

In social choice literature the single-winner voting rules can be categorized into the scoring methods, e.g., Borda, which are based on scoring protocols and the Condorcet consistent rules, e.g., Copeland, which are based on pairwise comparisons. Condorcet is considered as the founding father of social choice theory and proposed the following rule. An alternative x is said to beat alternative y in a pairwise comparison if the majority of the agents prefer x to y, i.e., rank x above y. An alternative that beats every other alternative in a pairwise comparison is the *Condorcet winner*.

The social's choice system projected architecture allows the decision maker to choose if she wants the outcome to be a full ranking of the alternatives or a set of "best" alternatives. Using a classic voting method can handle only the case of aggregating the individual preferences to a full ranking but does not cover the case of finding the set of equally winning alternatives, i.e., the "best" alternatives. Hence, we refer to multi-winner voting rules that permit the fair aggregation of the individual preferences when selecting committees. A committee is a fixed-size subset of alternatives who are equally winners. The goal of these rules is that every agent is represented by a committee member and thus these methods are appropriate for electing the best k alternatives, when k is the number of alternatives in the committee. Contrary to the data collecting subsystem which is specific for each survey, the voting subsystem is generic tool which can be used for all the use-cases.

Argumentation subsystem: The role of this subsystem is to provide the set of coherent and complete points of view. In order to achieve that we use the justifications of the rankings on alternatives/criteria of the different agents and eliminate through logical argumentation the inconsistencies between the agents' preferences. Hence, a deliberation phase interacts with the voting and the ambiguous preferences are eliminated from the preferences of the agents. Following this procedure, we build the justified preference profile, which can serve as input to the voting subsystem.

Decision subsystem: The role of this subsystem is to provide a recommendation for the decision maker on the given decision problem which is the final output of our software. The recommendation we provide is the parameterized (by the decision maker) outcome of the voting and/or the argumentation subsystem.

4.1.4 Social Choice system

The social choice system is one of the two systems that compose the decision-making tool and can act independently or in combination with the deliberation system. The system's role is to provide a recommendation that reflects the socially fairest solution. The system provides a quantitative approach and selects the best alternatives with regard to the aggregation of the

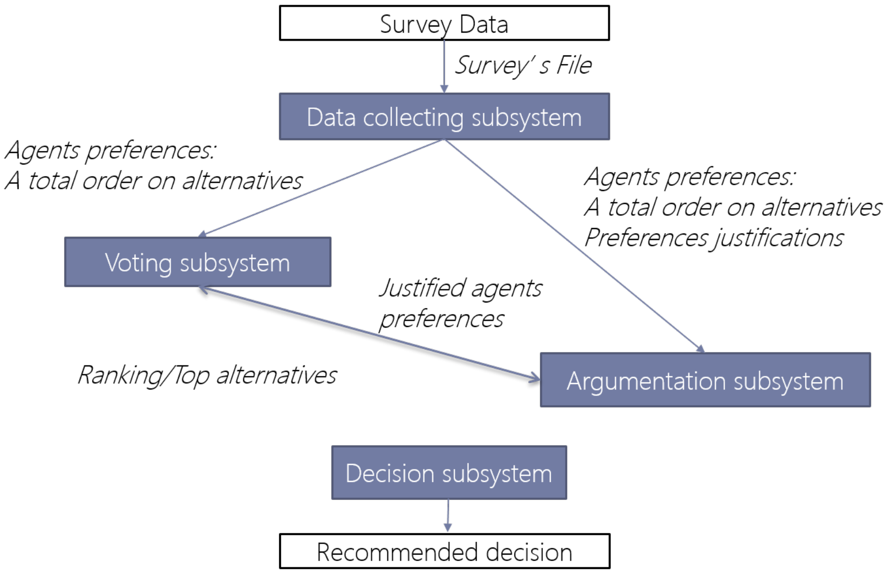


Figure 4.1: The architecture of the decision support software in high-level.

individual preferences, but if the decision maker demands a more qualitative approach then she has to take into account the deliberation phase, hence the design of the deliberation system which complements this one. The social choice system is composed of an implementation of the following three subsystems: the data collecting, the voting and the decision subsystems. The general architecture of our design defines that the communication between the data collecting and voting subsystems is based on files since a structured json file is the input for the voting subsystem. On the contrary, the voting and decision subsystems communicate internally between the Java code. In this section we present our implementation and the functionality of the social choice system. The tool is implemented in Java and we use the Swing toolkit for creating the graphical interface.

Implementation of the data collecting subsystem

The role of the data collecting subsystem is to "transform" the raw data which is extracted from different surveys into the suitable input's format for the voting subsystem, i.e., structured information for each one of the questions/decision problems of the survey. Recall that each survey provides us data which is related to the preferences of the agents in an abstract format. Therefore, the mandatory structured information includes the list of the alternatives and the agents' rankings on them. In order to fix the structured information, we construct a structured "json" file format with specific attributes that reflects the format of the survey and hence the voting subsystem can process this file as input. Remember that raw data can have different formats depending on the format of the surveys. Hence, we implement in our code several functions that can handle several different export formats of raw data coming from LimeSurvey (LimeSurvey GmbH, 2003). Their functionality is to transform the different formats into a unique structured data format, and hence to the structured json file. We are implementing the transformation from LimeSurvey, since it is a popular web server-based software which enables users to develop and publish on-line surveys and collect responses, using a user-friendly web interface. The structured json file is designed in a way to correspond to the structured data of the input of the voting subsystem and includes all the information of the surveys. To this end, we standardize the format of the input of the voting subsystem (and hence the json file) in case of future addition of new data input. The structured form of the voting input permits the implementation of additional functions transforming from different surveys formats to the json file. Therefore, the design and the implementation of the data collecting subsystem allows the social choice system to have the capability of including new input formats as long as classes and functions are implemented that can handle the transformation of different file formats to the json standardized input. We have also implemented a direct communication between the data collecting and voting subsystems. This extra feature is directly implemented in the data collecting subsystem and the transformation from the csv file to the voting's subsystem input is done internally. One should note that this is not our general approach but is included in order to facilitate the user and process directly the LimeSurvey's csv file.

Implementation of the voting subsystem

The basic functionality of the voting subsystem is to aggregate agents' preferences and compute the socially fairest alternatives. To achieve that we create a Java class which implements the subsystem's features. We describe them in the reminder of this section.

Input type. There are two input types for the voting subsystem. The default file input's format is a structured json file containing information about the different decision problems and the agents' individual preferences. A single json file can support more than one decision problem. A decision problem is defined by the topic the decision maker designates. Hence, each decision problem corresponds to a question that defines a poll. Therefore, we implement the json file to include several decision problems when multiple questions on the same set of agents are included in the survey. For each question we include in the json file the following information:

- "Question id": A unique number for identifying between the different decision problems.
- "Poll's Question": Contains the formulation of the decision problem in an interrogative form as it is imposed by the decision maker.
- "Alternatives": The list of the alternatives that correspond to the specific decision problem.

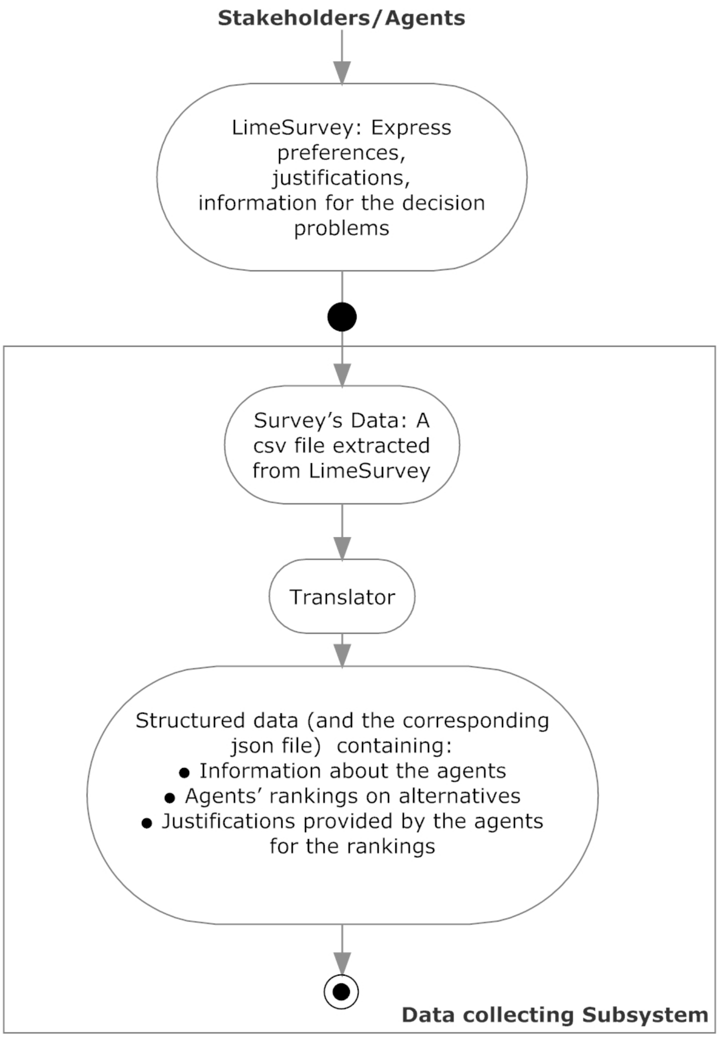


Figure 4.2: The activity diagram of the data collecting subsystem.

- "Agents characteristics": Contains the categories on which the agents can be classified to. The set of agents can be classified into different categories according to their characteristics. In this field the decision maker defines all the characteristics that she wants to be taken into account for classifying the agents. E.g., one category can be "age".
- "Categories of agents": Contains the sub-categories of the above categories on which the agents can be classified to, e.g., for category "age", we can have the following sub-categories: "$18 - 25''$, "$26 - 35''$, "$36 - 49''$, "$> 50''$.

For each agent the following information is stored in the json file:

- "Agent id": A number for identifying between the different agents.

- "Values for agents characteristics for Question id x": Contains the personal information (values) for each agent corresponding to the categories defined in the decision problem x.
- "Alternatives ranking for question id x": Contains the total order of the alternatives according to the agent for problem x.
- "Justification for alternatives ranking for question id x": Contains the reasoning behind the individual preferences of the agent over the alternatives for problem x.

The second input file format is a csv file derived from LimeSurvey containing the above same information in different format. The first line of the csv file is composed of the information about the decision problems and then there is a line for each agent which represents the preferences and the information of this agent.

Aggregation of individual preferences. The core functionality of the voting subsystem is the aggregation of the agents' individual preferences to a collective preference. Our software proposes to compute the output aggregated ranking using the following voting rules keeping in mind to cover the whole spectrum of the literature: Borda, Kemeny-Young, k-approval, Extended Tideman's simplified Dodgson and Multi-winner STV. Therefore, we implement Borda rule ([43]) as an example of a scoring rule and the Copeland's method ([41]) as a representative of Condorcet-consistent rules. We also implement k-Approval ([30]) because we want to cover the case where the preferences of the agents include a set of equally "accepted" set of alternatives instead of a total order on them. We also implement the Extended Tideman's simplified Dodgson rule ([37]) from the new generation of rules that were deployed recently in the Computational Social Choice literature to overcome the computational complexity issues of the other rules. This rule has the advantage of being polynomial time and thus can be computed on any input size. In addition, we propose a multi-winner voting rule, i.e., the multi-winner STV, in order to aggregate the agents' individual preferences to a set of equally winning alternatives. A definition of the implemented rules follows.

BORDA'S COUNT: It is a scoring based voting rule. It is one of the simplest and most intuitive scoring rules, where each alternative receives from 0 to $m-1$ points from each agent, where m is the total number of alternatives. The protocol is that the alternative which is ranked in the k-th position by agent i receives $m-k$ points from this agent. The total Borda score of each alternative is the accumulation of all the points received from the agents. The alternative with the highest accumulated score wins and the rest of the alternatives are ranked according to their score.

K-APPROVAL: Each agent "approves" (i.e., selects) k out of m alternatives and each one of them gets 1 point. The approval score of an alternative then is the cumulative number of the points she receives from all the agents. In all approval rules (including plurality and veto) the alternative with the highest score wins and the rest of the alternatives follow according to their score. For $k = 1$ we obtain the Plurality rule, which is one of the simplest and most fundamental rules in the history of social choice. Also, for $k = m - 1$ we obtain Veto where each agent approves all but one alternative. The intuition is that each agent poses her disapproval (veto) on one alternative.

COPELAND: In Copeland's rule alternatives are compared pairwise. When an alternative is preferred by the majority of the agents she receives one point and the other alternative receives zero points. In case of a tie, in the classical version of Copeland's rule both receive 0.5 points. The sum over the points is called the Copeland score. Winner(s) are the alternatives with maximum Copeland score and the rest follow according to their score.

EXTENDED TIDEMAN'S SIMPLIFIED DODGSON RULE: This rule depends also on pairwise comparisons between alternatives and is based on the well-known voting rule of Dodgson ([26]). Since Dodgson's rule is hard to compute we choose this rule because it is a Dodgson's approximation algorithm which can be efficiently computed. This rule is an extension of Tideman's rule and the score of each alternative x is defined as follows. If an alternative is a winner under the Condorcet method then her score is 0, otherwise her score is computed by the following formula:

$$\mathrm{sc}_{\mathrm{Td}'}(x) \quad = \quad m \cdot \mathrm{sc}_{\mathrm{Td}}(x) + m(\log m + 1).$$

where m is the total number of the alternatives and

$$\mathrm{sc}_{\mathrm{Td}}(x) \quad = \quad \sum_{y \in Alts. \setminus \{x\}} \max\{0, k - l\}.$$

where k is the number of agents that prefer y over x and l is the number of agents that prefer x over y. The alternative with the minimum score wins and the rest of the alternatives are ranked according to their score.

MULTI-WINNER STV: This rule belongs to multi-winner voting rules which are designed for selecting a fixed k-sized committee of alternatives, i.e., the set of k-winning alternatives. Hence, multi-winner STV is the most appropriate rule to be used when the output selection is set to k-top alternatives. The Single Transferable Vote rule (STV) executes a series of iterations, until it finds k winners. A single iteration operates as follows: If there is at least one alternative with Plurality score at least $q = \lfloor n/(k+1) \rfloor + 1$, then an alternative with the highest Plurality score is added to the committee; then q agents that rank her first are removed from the election (a randomized tie-breaking plays an important role here), and the selected alternative is removed from all agents' preference orders. If there is no such alternative, then an alternative with the lowest Plurality score is removed from the election (again, ties are broken uniformly at random). The Plurality scores are then recomputed.

As noted previously, there is no "best" voting rule and on the same input, i.e., the preference profile of the agents, different voting rules can compute different aggregated rankings as output, leading to the need, for the decision maker, to be able to choose the voting rule used for computation. In the following example we are showing that phenomenon.

Example. Let us consider the following individual preferences for the agents. We have 55 agents with the following rankings. We should note here that $x > y$ indicates that an agent prefers x to y.

- 18 agents: $A > D > E > C > B$
- 12 agents: $B > E > D > C > A$
- 10 agents: $C > B > E > D > A$
- 9 agents: $D > C > E > B > A$
- 4 agents: $E > B > D > C > A$
- 2 agents: $E > C > D > B > A$

We are now applying the following different voting rules and obtain the corresponding results.

- Borda: Alternative D wins with $(18 \cdot 3) + (12 \cdot 2) + (10 \cdot 1) + (9 \cdot 4) + (4 \cdot 2) + (2 \cdot 2) = 136$ points. A gets 72, B101, C107 and E134.
- 1-approval (plurality): Here, A wins, with 18 votes. Alternatives B to E have score $12, 10, 9$ and 6 respectively.
- For Tideman's and Copeland's rules we observe that a Condorcet winner exists so she is also the winner under these rules. Alternative E is the Condorcet winner since she beats each of the other four options in pairwise comparisons, i.e., E beats A 37-to-18, B 33-to-22, C 36-to-19, and D 28-to-27.
- For Multi-Winner STV suppose that we want the best 3 alternatives, so we fix $k = 3$. A has plurality score of 18 which is at least $\lfloor 55/(3+1) \rfloor + 1 = 14$, so A is included in the winning alternatives and the 18 agents that rank A first are removed from the election, and also A is removed from all agents' preference orders. B has plurality score of 12 which is at least $\lfloor 37/(3+1) \rfloor + 1 = 10$, so B is also included in the winning alternatives and the 12 agents that rank B first are removed from the election, and also B is removed from all agents' preference orders. C has plurality score of 10 which is at least $\lfloor 25/(3+1) \rfloor + 1 = 7$, so C is also included in the winning alternatives. Hence, A, B and C are the winners.

Incomplete (truncated) preferences: It is the case where agents do not provide a complete ranking of the alternatives. This can happen when each agent is likely to know who her most favorite alternatives are but she is unwilling to put the effort into ranking the remaining ones. Then, it is safe to assume that she likes them less than the ranked ones but among the unranked ones there is no difference in preference. We use this assumption when computing the collective preferences under incomplete preferences. Therefore, the typical voting rules, like the abovementioned, have to be adjusted to cover this case. For example, for Borda, we could assume that each unranked alternative receives 0 points from a given vote (a method used in Slovenia, which is called pessimistic scoring model). Another method for Borda is the optimistic scoring model, where if there are m alternatives but a vote ranks only k of them, then the ranked alternatives get $m-1, \ldots, m-k$ points (depending on their position in the ranking) and each unranked alternative gets $m-k-1$ points. We will use the pessimistic model in our computation for Borda because it is more widely used. For k-approval if the preferences of the agents are incomplete but more than k alternatives are ranked then there is no difference and on the contrary when less than k alternatives are ranked by the agent, then all get one point. For the rules based on the pairwise comparisons we assume that if an alternative is ranked and the other is not, then the winner is the one ranked and when both are unranked we do not take into account this comparison.

Implementation of the decision subsystem

The objective for the decision subsystem is to provide a recommendation for the decision maker. The output (decision) will be based on the aggregation of the individual preferences but may contain different results according to the parameters that are imposed by the decision maker. To achieve that we create a Java class which implements different features, which we call decision-aiding features. There is a link in the implementation of voting and decision subsystems since decision-aiding and voting features interact. The decision maker can choose amongst decision-aiding features that can be classified into 3 categories according to the computation phase in which they belong to.

Voting phase. The first parameter on which the decision maker can base her decision is the choice of the appropriate voting rule. The decision maker can therefore choose one or more of the abovementioned rule(s) according to her needs and obtain different decisions, since different rules on the same preference profile can lead to different collective rankings.

Aggregation phase. During the decision-making procedure choosing the right voting rule is not the only crucial matter in order to support a decision. The decision maker is provided with different aggregation procedures that can help her take a better decision according to the needs

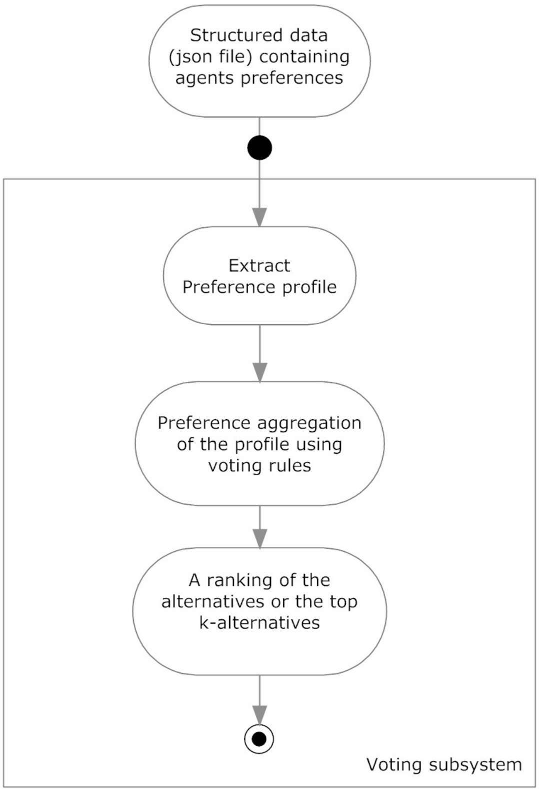


Figure 4.3: Activity diagram of the voting subsystem

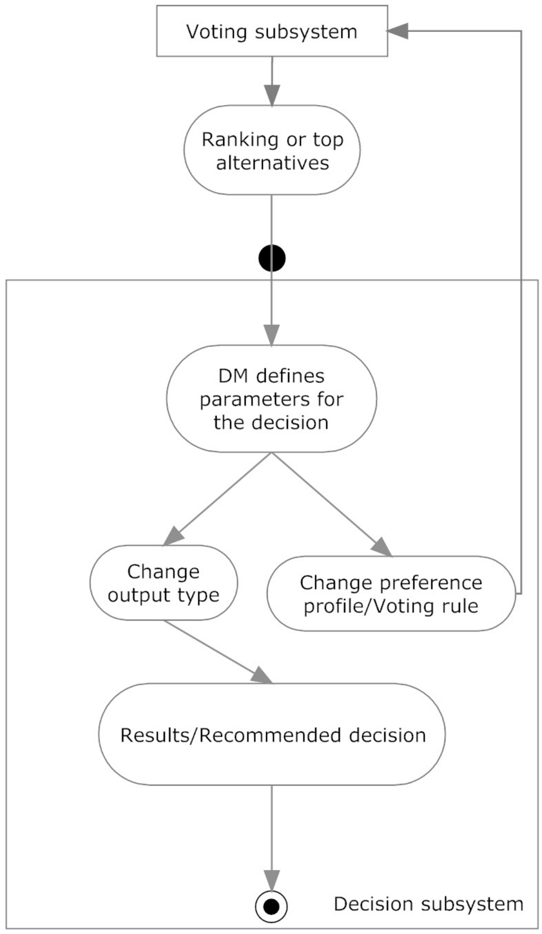


Figure 4.4: Activity diagram of the decision subsystem

of the problem by either taking a subset of the agents preferences or/and increasing the weight of the opinion of some agents. The preference profile can be partitioned in different subsets when agents share common characteristics. When agents belong to the same subset we say that they belong to a specific category, e.g., agents can be partitioned according to sex and hence we have two categories, i.e., male and female. It is therefore the preference profile which is taken as the input the one that differentiates the aggregation procedures. The decision maker can also combine categories in order to get the preference profile for the input (combination of categories). E.g., the decision maker can choose her input to be men aged 18-35. To get the desired input, the software allows the option to combine categories "Sex: malw" and "Age: 18-35". Therefore, we propose the following decision-aiding aggregation procedures.

TOTAL AGGREGATION: The decision is computed taking into account all agents' preferences.

AGGREGATION PER CATEGORY: In this procedure the agents are forming groups (subsets) according to their classification on different categories. Therefore, we apply a voting rule having as input the preference profile that corresponds to agents belonging only on the specified, by the decision maker, category (or combination of categories).

2-PHASE AGGREGATION: Also, in this procedure the agents are forming groups according to their category. In the first phase a ranking for each one of the agents' categories (subsets) is computed by applying a voting rule. In the second phase the decision is the collective preference computed by the aggregation of the rankings produced during the first phase. For both steps we are using the same rule. We consider that each one of the agents" types has equal say (i.e., weight) to the collective preference.

2-PHASE AGGREGATION WITH WEIGHTS: It is similar to the above procedure but in this case we do not consider that each one of the agents' types has equal say to the final collective preference. Instead we provide the decision-maker the ability to define the weights for the different categories according to the needs of the decision problem. The final ranking is computed by the proportional, according to the weights, aggregation of the votes of each category.

Output phase. During this phase the decision maker can base her decision on different types of output. The output produced by the tool can have the following forms.

SINGLE WINNER: The recommendation that is provided to the decision maker is the winning alternative according to the specified rule. In case of a tie we can have multiple winners.

K-TOP ALTERNATIVES (COMMITTEE SIZE): The recommendation is a committee of size k, which corresponds to the set of the best k alternatives. A multi-winner voting rule is used.

RANK: The recommendation is provided in the form of a complete ranking over the alternatives.

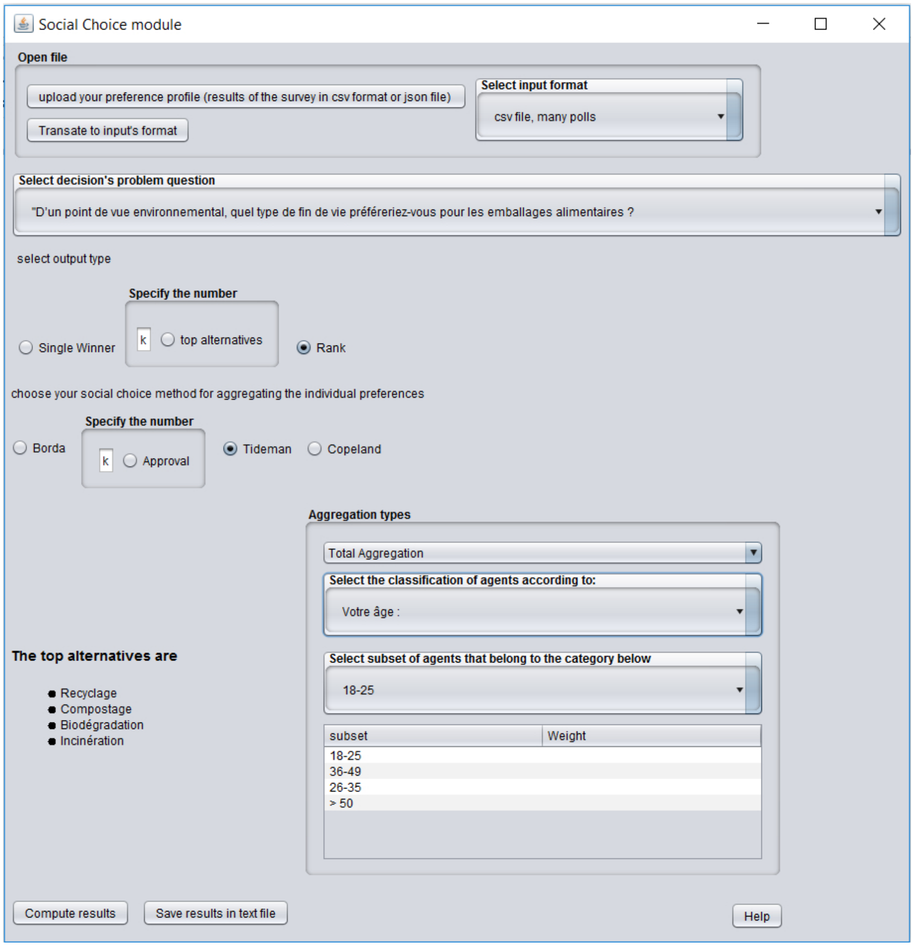


Figure 4.5: A screenshot of the GUI.

Graphical User Interface.

The goal of our decision support software is to be used by decision-makers which are non-voting experts. Taking this goal into account, we implement a graphical user interface (GUI) which corresponds to the functionalities described in the subsystems. We believe that the GUI is user-friendly and permits the decision maker to explore all the functionalities in terms of preference aggregation and achieve a decision based on her needs. A screenshot of the GUI is shown in figure 4.5.

The usage of the tool can be seen through the layout of the interface which is the following. On the upper part of the application we depict the data collecting subsystem where the user can load the survey"s file (csv) or the voting input file (json). Right beneath is the translator where one can transform the survey csv file into the json file format which is appropriate for input to the voting subsystem. In the middle part of the GUI the voting subsystem is being visualized where the decision maker can choose the voting rule that is going to be used for the aggregation of the preference profile. The decision maker has the option to choose among different rules and may obtain different results. Then in the lower two panels the decision subsystem is being shown, where the decision maker can choose the aggregation and output types. In order to select the aggregation types a new window appears (figure 4.6). In this example the decision maker takes into account only the agents aged $26-35$.

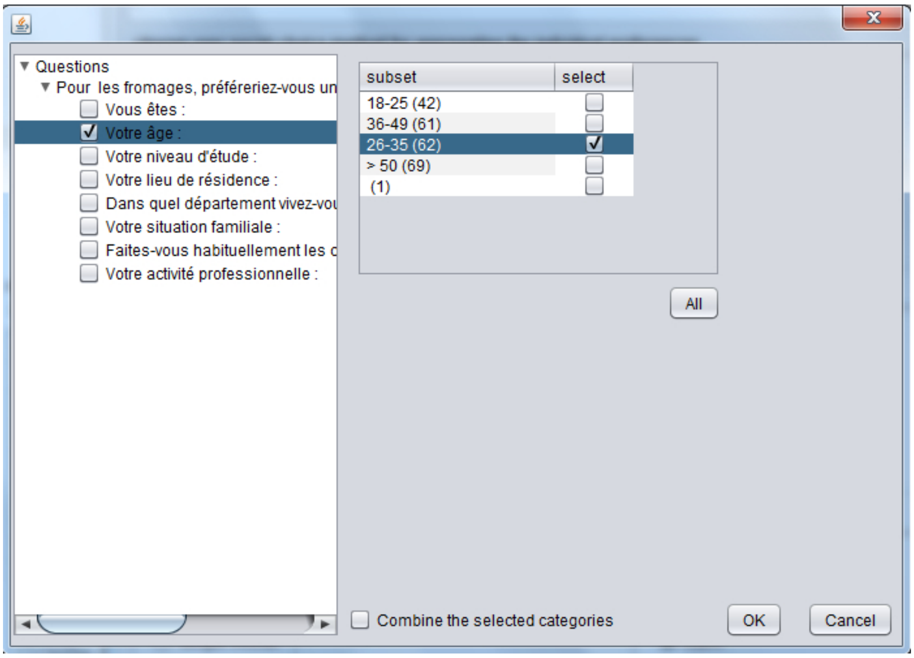


Figure 4.6: Interface for aggregation types.

The output of the computation is depicted in a text form. The decision maker can choose through the interface the parameters she wants to impose and hence obtain the corresponding results. The subpart of the interface for the results is depicted in figure 4.7. We also provide the option to save the results in a text file.

4.1.5 Examples on real data: Packaging choice decisions for Ecobiocap and opinion analysis

In order to demonstrate the usability of our tool and validate the presented methodology we study real-data decision problems on food packaging selection. The studied decision problem aims at recommending relevant alternate food packagings based on multiple criteria. One critical factor in recommending new packagings is the opinion of the consumers. Therefore, we apply our tool to this problem and obtain the preferences of the agents from a case study which is extracted from a survey that was conducted in a national level for France for the needs of the

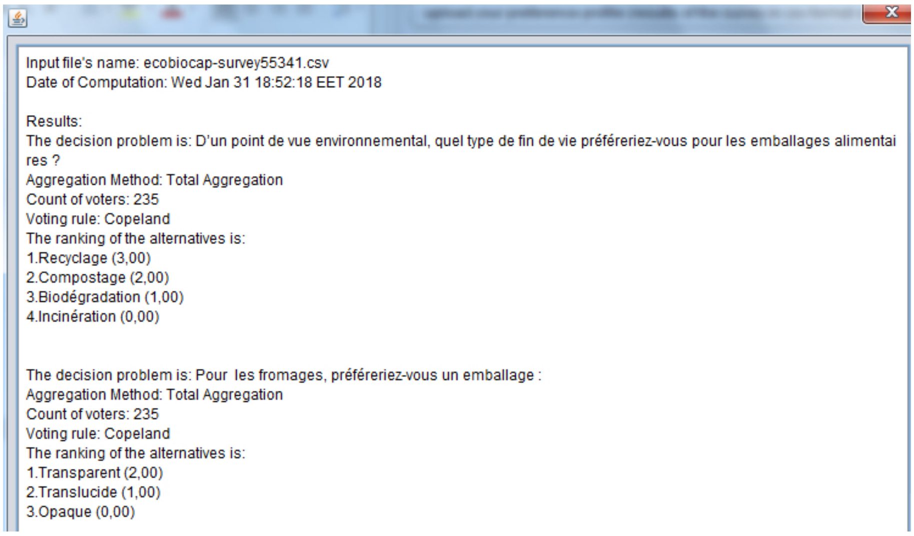


Figure 4.7: Results interface.

Ecobiocap (ECOefficient BIOdegradable Composite Advanced Packaging) project. This survey is related to different functionalities regarding food packaging where 235 citizens helped by answering the questions. Therefore, we use this survey for collecting the data, i.e., the citizens (agents) preferences and the justifications, and answering in the next step the decision problem. One should observe that using our applied procedure on these examples is not restrictive on the applicative usage of our software since it is designed as a more general framework which can support future possible applications and problems.

The first category of problems included in this survey consists in finding the best alternatives for food packaging on different kinds of foods. Hence, the citizens were asked to give their preferences on different questions regarding cheese, fruits & vegetables and sandwich packagings. These questions correspond to different decision problems (for each type of food) as the agents evaluated and ranked the alternatives for each question separately. The alternative options given to be ranked by each agent were transparent, translucent and opaque.

The second decision problem on which the agents were asked to give their opinion concerns the type of the material for food packaging. Therefore, each agent gave her preference on the type of the material she prefers for producing the food packaging. The agents had to choose between two alternatives for this problem, i.e., bio-source (in particular, using agricultural by-products to produce the packaging) and petro-source.

For the next decision problem each agent had to answer if she approves the usage of clay nanoparticles for the composition of the food packaging. The agents had to choose between yes and no.

The last decision problem is related to the end-of-life management of the food packaging. The agents gave their preferences on the type of the end-of-life they prefer for the packagings. For this problem the set of alternatives were: recycling, composting, biodegradation and incineration.

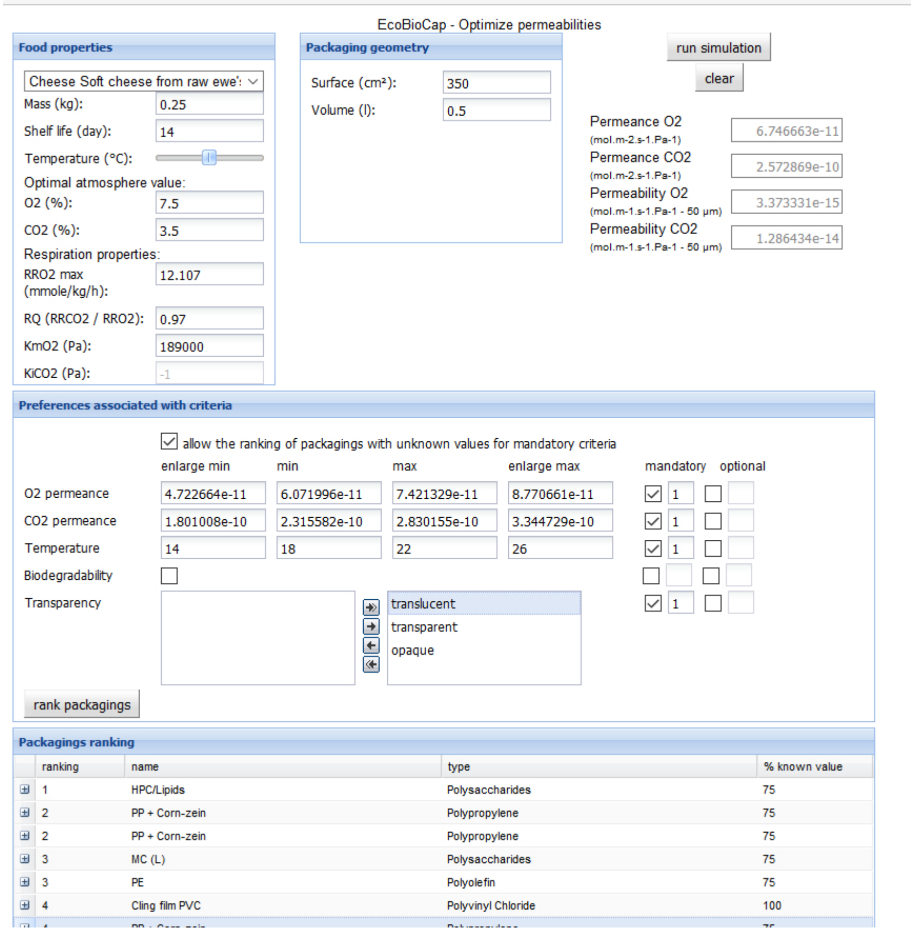


Figure 4.8: Multi-criteria decision support (MCDSS) results interface.

Preference aggregation for Ecobiocap's MCDSS. In this application, the social choice system computed aggregated preferences associated with different populations for specific food packaging characteristics. Then, those aggregated preferences have been used as input parameters for a multi-criteria decision support system - called MCDSS ([45]; [59]) which retrieves the most relevant packaging for a given food. In the following, we present the results obtained for cheese packaging regarding transparency. Using individual preferences expressed by 235 citizens in the Ecobiocap survey, our tool has computed the following aggregated preferences:

- Case 1: French population as a whole (235 votes): *transparent* > *translucent* > *opaque*
- Case 2: people living in Ile de France (46 votes): *translucent* > *transparent* > *opaque*

The MCDSS has been used twice to retrieve the most relevant packagings to pack a soft cheese from raw ewe"s milk (250 grams, shelf life of 14 days, storage at ambient temperature) using preferences associated with populations of cases 1 and 2. For Case 1, the best packaging is a MC(L) Polysaccharide as for Case 2, the best one is a HPC/Lipids Polysaccharide. The complete list of most preferred packagings for Case 2 is presented in the "Packaging ranking" window in the lower part of figure 4.8.

Opinion analysis as a powerful tool for decision-making. Another possible use of the system is to provide recommendations to the decision maker for designing potential food packagings taking into account the citizens opinion.

The results revealed that for the type of packaging the citizens prefer for cheese, fruits & vegetables and sandwich the transparent is ranked higher than translucent while opaque is the last one in all but the plurality rule. In the case of 2 or more alternatives plurality is not considered an appropriate method by social choice theorists. Hence, the recommendation provided to the decision maker is than citizens prefer transparent to translucent to opaque.

Regarding the second decision problem, the results revealed that in the collective preference of the citizens for the type of the material they prefer for producing the food packaging the bio-source is ranked higher than petro-source. The same result appeared when all the voting rules were used for the aggregation of their individual preferences. Hence, the recommendation provided to the decision maker is than citizens prefer bio-source to petro-source.

For the decision problem concerning the usage of clay nanoparticles for the composition of food packaging, we used plurality and the recommendation provided to the decision maker is than citizens do not prefer the usage of clay nanoparticles.

The results for the last decision problem, which corresponds to the end-of-life for packagings, revealed the following same ranking for the rules of Borda, Tideman's and Copeland: recycling, composting, biodegradation and incineration. Using plurality we obtained a slight different ranking: recycling, biodegradation, composting, and incineration but observe that the winner is still the same under all voting rules. In this case we have four alternatives so we will not use the ranking provided by plurality and hence, the recommendation provided to the decision maker is than citizens prefer recycling to composting to biodegradation to incineration for the end-of-life for food packagings.

Summing up the results of all the above decision problems the decision provided by the collective preferences of the citizens is that they prefer a food packaging that satisfies the following features. It should be transparent, made of bio-source materials without the use of clay nanoparticles and should also be recyclable.

Also, based on the decision maker parameters the following interesting insights appeared when using different voting rules and aggregation types for the aggregation of the individual preferences. The first category of the observed insights in real data examples concerns the discrepancies for the same preference profile when a different voting method was used. For example, as already mentioned the case of plurality compared to the other voting rules. The second category of insights observed are the ones related to the aggregation when different subsets of citizens (i.e., partitions of the preference profile) were taken into account. We make the following interesting observations. Note that all the remarks that we make below refer to computation under the same voting rule.

Observation 1. For the problem of the end-of-life, the collective ranking according to different sex is different when men are compared to women. The collective preference of the 91 men included in the survey is: recycling, biodegradation, composting and incineration. On the other hand 143 women voted for the survey and their collective ranking is: composting, recycling, biodegradation and incineration.

Observation 2. For the same problem different results are also obtained if we change the category of aggregation to age. All the age groups except the ones over 50, i.e., "$18-25$", "$26-35$", "$36-49$", have the same ranking: recycling, composting, biodegradation and incineration while people over 50 have composting, biodegradation, recycling and incineration.

Observation 3. The end-of-life problem is not the only one with interesting insights. For the problem of cheese packaging we also obtain different rankings when different subsets of the preference profile are used. For example, citizens living in the Herault department have the following preference: transparent, translucent, opaque while people living in Paris have: translucent, transparent, opaque.

Therefore, we can see in practice that collective preferences change according to a partition of the preference profile used as the input. Hence, we provide the partitioning of the preference profile as an important tool to the decision maker in order to support her decision when the nature of the decision problem is driven by different constraints. For example, if the decision maker is only interested for producing a cheese packaging that will only be sold in Paris then she should rely her decision only on the preferences of the agents that live in Paris (observation 3). We believe also that the 2-phase aggregation is a really important tool for the decision maker as we state in the next observation.

Observation 4. For the end-of-life problem we can see that people living in Herault have different preferences from people living in Paris. People from Herault prefer $recycling > composting > biodegradation > incineration$ while people from Paris equally prefer *composting* and *biodegradation* to *recycling* and *incineration*. The total of aggregation of the all the agents produces the collective ranking: *recycling* > *composting* > *biodegradation* > *incineration* which is the same as the Herault's collective ranking. This is due to the ratio of the statistical sample which is not proportional to the real population ratio because in the survey we had 137 agents voting from Herault and 14 from Paris. However, if the decision maker wants a fairest solution according to the population ratio she can adjust the weights of the agents living in different regions. Hence, if we assume weight of 12 to Paris compared to 1 for Herault (12 million inhabitants versus 1 million) then the following collective ranking is computed: *composting* > *biodegradation* > *recycling* > *incineration*. Note that, we also assign zero weight to the other departments since the statistical sample is small for them.

4.1.6 Conclusion and Future work

In this paper, we have proposed a software tool for decision-making for issues coming from agricultural engineering with the aid of computational social choice and argumentation framework. We describe thoroughly the social choice system of the decision support software and demonstrated its application on a real-data example in the context of Ecobiocap project where the objective is to find the best food packaging in terms of material, visibility and end-of-life.

As future work, we want to further extend our research towards the integration of argumentation and social choice for "better" decisions. Hence, our goal is to design and implement the deliberation system which will complete the decision support software.

4.2 Implementation of algorithms for collective decision-making employing reasoning based aggregation methods

4.2.1 Introduction

The current section is dedicated to the presentation of the functionality of the algorithms that were used to support the theoretical results of this research. The basic functionality of these algorithms is twofold. First, they deal with the computation of the extensions of the argumentation frameworks that are build from agents preferences. Second, they perform a computation of the aggregation of the preference profile based on social choice methods. The nature of these algorithms will permit us to use on future work regarding implementation of decision making software tools that are based on reasoning based aggregation methods. The following algorithms are implemented in Java, a well-known and generic-purpose programming language.

4.2.2 Implemented algorithms and methods.

The basic role of the developed software is to compute the winner or a ranking of the alternatives (i.e., the output) in a collective decision problem expressed by "voting polls" by taking into account the agents' preferences (i.e., the input). Those preferences refer to criteria that have been set in advance for each voting poll and can be expressed by the agents in 2 ways. The first way is a cardinal method where each agent gives a score on a scale from 1 to 10 reflecting her preference on each alternative for each criterion. The second way is an ordinal method where each agent defines a ranking (total order) of the alternatives for each criterion. For each criterion a "weight" is defined which states the relative importance of the criterion for computing the collective decision, i.e., the winning ranking of the alternatives. The data containing the agents' preferences are stored in a text file that has a specific structure. We have also implemented a "random" data generator for producing "random" preferences by using one of the pseudo-random number generation algorithms. The goal is to compare the different ways of computing the ranking order of the alternatives when implementing a large number of voting "polls".

The implemented methods for computing the ranking order are based on argumentation framework and social choice theory. The comparison among the methods is done via classical methods from voting theory,i.e., Borda, Condorcet, etc. In general, the computation is done in 2 phases (steps). In the first phase we construct an argumentation framework from the agents' preferences, i.e., we use agents' preferences to build the arguments and the attacks between them to best reflect the agents' preferences. This can be done with different rules that are defined below. The outcome of an argumentation framework leads to the computation of the extension(s), which is (are) a set of coherent arguments that reflect a possible viewpoint of the aggregated agents' preferences. In the second phase the extension(s) are being interpreted in a way defined in each method and the new "justified preference profile" is created, i.e., a profile that includes the "justified preferences" of the agents. The preferences are now justified as they are the outcome of the argumentation phase and have been generated by the first phase of our algorithm. The winner or the winning ranking is either a direct outcome of the procedures from the argumentation theory or the outcome of Voting theory methods, where the argumentative outcome, i.e., the justified preference profile is reported to a voting rule to declare the ranking.

Functionality of the implemented algorithms and methods.

1. The agents report their preferences using the first method (cardinal). The agents' arguments are binary, i.e., positive or negative, and of type $(criterion, alternative)$. The classification of an argument to positive or negative is done according to the preference value. If $value$ is over 5.5, i.e., $(5,5 = (1+10)/2)$ then the argument is characterized as positive, else negative. For each alternative we pairwise compare the criteria and the positive criterion defeats the negative one. Hence, arguments $(alternative, criterion)$ are created and the attack means that the 2 compared criteria for the alternative are opposite. We compute this condition for all the agents, i.e., an argument is positive if

the majority of agents rank alternative in the specified criterion with $value > 5.5$. For example, given that argument 2_c is positive, i.e., the majority of agents rank alternative c in the second criterion with $value > 5.5$ and that argument 4_c is negative, then we create an attack $2_c \rightarrow 4_c$. Then, we use the *grounded* semantics to compute the outcome of the framework and let for example, the result to be the single extension $[1\_d, 1\_c, 1\_b, 2\_b, 3\_c, 2\_c, 4\_d, 2\_d, 6\_a, 4\_a, 6\_c, 5\_b, 5\_a, 5\_c]$. Therefore, in criterion 1 the "winning" alternatives are d, c, b, in criterion 2 alternatives b, c, d , in 3 alternative c, etc. Taking into account the extension and according to the criteria weight matrix a grade for each alternative is computed (using weighting sum, but also possible to use other methods). Then we create a new framework where for each argument included in the extension an argument of type (*alternative*) is created, where $alt1 \rightarrow alt2$ if $grade(alt1) > grade(alt2)$. The final extension of this framework is the winner, e.g., c.

2. The agents report their preferences using the first method (cardinal). The agents' arguments are binary, i.e., positive or negative, and of type $(criterion, alternative)$. The classification of an argument to positive or negative is done according to the preference value. The *value* is equal to the arithmetic *mean* of the grades given by all agents on each specific pair (criterion, alternative) Hence, for $value > mean$ the argument is characterized as positive, else negative. For each alternative we pairwise compare the criteria and the positive criterion defeats the negative one. This method is similar to method 1 with the difference that the computation of arguments is not done for all agents together but for each one separately. For example, given that argument 2_c is positive, i.e., at least one agent ranks alternative c in the second criterion with $value > mean$ and that argument 4_c is negative, then we create an attack $2_c \rightarrow 4_c$. Then, we use the *grounded* semantics to compute the outcome of the framework and let for example, the result to be the single extension $[5\_b, 5\_c, 2\_b, 3\_c, 2\_c]$. Taking into account the extension and according to the criteria weight matrix a grade for each alternative is computed. Then we create a new framework where for each argument included in the extension an argument of type (*alternative*) is created, where $alt1 \rightarrow alt2$ if $grade(alt1) > grade(alt2)$. The final extension of this framework is the winner, e.g., c.

3. The agents report their preferences using the first method (cardinal). The agents' arguments are of type $(criterion, alternative)$. For each criterion the alternatives are pairwise compared and an attack is formed between the arguments when the majority of agents prefer one alternative to the other, i.e., $(criterion, alt1) \rightarrow (criterion, alt2)$ when the majority of agents prefer in criterion *criterion* alternative $alt1$ to $alt2$. Then, we use the *grounded* semantics to compute the outcome of the framework and let for example, the result to be the single extension $[7\_b, 6\_a, 1\_d, 7\_c, 5\_c, 7\_d, 3\_c, 2\_c, 4\_d]$. Taking into

account the extension and according to the criteria weight matrix a grade for each alternative is computed. Then we create a new framework where for each argument included in the extension an argument of type ($alternative$) is created, where $alt1 \rightarrow alt2$ if $grade(alt1) > grade(alt2)$. The final extension of this framework is the winner, e.g., c.

4. The agents report their preferences using the first method (cardinal). The agents' arguments are of type $(criterion, alternative, agent)$. For each criterion and agent the alternatives are pairwise compared and an attack is formed between the arguments when an agent prefers one alternative to the other, i.e., $(criterion, alt1, agent) \rightarrow (criterion, alt2, agent)$ when the agent has graded in criterion $criterion$ alternative $alt1$ with higher grade than $alt2$. Then, we use the *grounded* semantics to compute the outcome of the framework, which is collective since it includes all the attacks created from all the agents and let for example, the result to be the single extension $[3\_c\_3, 3\_c\_2, 3\_c\_1, 1\_d\_2, 1\_d\_1, 4\_d\_3, 5\_b\_2, 5\_c\_3, 4\_d\_1, 5\_c\_1, 4\_b\_2, 6\_a\_1, 1\_c\_2, 1\_b\_2, 1\_d\_3, 7\_d\_2, 7\_c\_3, 2\_c\_2, 2\_c\_1, 6\_a\_3, 2\_c\_3]$. From this extension we derive that in criterion 3 alternative c beats the others in agents $1, 2, 3$. Taking into account the extension and according to the criteria weight matrix a grade for each alternative is computed. Then we create a new framework where for each argument included in the extension an argument of type ($alternative$) is created, where $alt1 \rightarrow alt2$ if $grade(alt1) > grade(alt2)$. The final extension of this framework is the winner, e.g., c.

5. The agents report their preferences using the first method (cardinal). The agents' arguments are of type $(criterion, alternative)$. The attacks are formed with a combination of two criteria, which we have already described above. First, we create attacks like method 1, i.e., for each alternative we pairwise compare the criteria and the positive criterion defeats the negative one (like method 1). Then, attacks are created like method 3, i.e., for each criterion we pairwise compare the alternatives and an attack is created when the majority of agents prefer one alternative to the other. Both kind of attacks are included in the framework and we use the *grounded* semantics to compute the outcome, which is collective since it includes all the attacks created from all the agents and let for example, the result to be the single extension $[7\_b, 6\_a, 1\_d, 5\_c, 3\_c, 2\_c, 4\_d]$. Taking into account the extension and according to the criteria weight matrix a grade for each alternative is computed. Then we create a new framework where for each argument included in the extension an argument of type ($alternative$) is created, where $alt1 \rightarrow alt2$ if $grade(alt1) > grade(alt2)$. The final extension of this framework is the winner, e.g., c.

6. The agents report their preferences using the first method (cardinal). The agents' arguments are of type $(criterion, alternative, agent)$. For each criterion and agent the alternat-

ives are pairwise compared and an attack is formed between the arguments when an agent prefers one alternative to the other, i.e., $(criterion, alt1, agent) \rightarrow (criterion, alt2, agent)$ when the agent has graded in criterion *criterion* alternative $alt1$ with higher grade than $alt2$. The attacks are created like method 4 but here, the framework is formed differently. In this case we do not construct a collective framework which includes all the attacks from all the agents but instead for each agent an individual framework is formed. Then, we use the *grounded* semantics to compute the outcome of the individuals frameworks and let for example, the result to be the single extension $[3\_c\_1, 6\_a\_1, 1\_d\_1, 4\_d\_1, 5\_c\_1, 2\_c\_1]$. Taking into account the extension (which refers to a single agent) and according to the criteria weight matrix a grade for each alternative is computed. The alternatives are ranked in terms of their grade and this ranking corresponds to the total order (vote) of the agent. We compute the rankings (votes) of all agents and this constitutes the "justified" preference profile. This profile is then reported to a voting rule (here, we use Borda's and Tideman's ones) to compute the final collective winning ranking.

7. The agents report their preferences using the second method (ordinal). The agents' arguments are of type $(criterion, alternative, position)$. Bidirectional attacks, (e.g., $arg1 \leftrightarrow arg2$) are formed in the following cases:

   - when in the same criterion an alternative is ranked in a different position in two agents.
   - when in the same criterion in two different agents two different alternatives have the same rank.

   Then, we use the *preferred* semantics to compute the outcome of the framework and let for example, the result to be the following extensions $[1\_b\_3, 1\_a\_2, 2\_a\_3, 1\_c\_1, 2\_b\_2], [1\_b\_1, 1\_a\_2, 1\_c\_3, 2\_a\_3, 2\_b\_2]$. The extensions correspond to virtual agents and for each one of them a grade for each alternative is computed by taking into account the criteria weight matrix. Hence, the virtual vote corresponding to the extension is a ranking of the alternatives according to their grade. In this way, we compute all the votes that constitute the "justified" preference profile. This profile is then reported to a voting rule (here, we use Tideman's) to compute the final collective winning ranking.

8. The agents report their preferences using the second method (ordinal). The agents' arguments are of type $(criterion, rank)$. Bidirectional attacks are formed when in the same criterion different agents give a different rank of the alternatives. Hence, in each criterion we pairwise compare the rankings given by each agent and if different then a bidirectional attack is formed for each pair. Then, we use the *preferred* semantics to compute the

outcome of the framework and let for example, the result to be the following extensions $[2\_acb, 1\_bca], [1\_cab, 2\_acb]$ where, in the example, $1, 2$ are the criteria and $acb, bca, etc.$ are the rankings. Ranking bca means that alternative b is ranked first, then c, and then a. The extensions correspond to virtual agents and for each one of them a grade for each alternative is computed by taking into account the criteria weight matrix. Hence, the virtual vote corresponding to the extension is a ranking of the alternatives according to their grade. In this way, we compute all the votes that constitute the "justified" preference profile. This profile is then reported to a voting rule (here, we use Tideman's) to compute the final collective winning ranking.

9. The agents report their preferences using the second method (ordinal). The arguments are of type $(criterion, rank)$ and 3 types of arguments are defined: preference relation, ranking, and blank argument(s). First, we create the preference relation arguments taking the preferences of all the agents. Here, the $rank$ is composed of two alternatives in which the first one beats the second. For example, argument $(criterion, r(a_i, a_j))$ means that in criterion $criterion$ there is an agent in which alternative a_i is ranked above a_j. Then, we form the ranking arguments where $rank$ corresponds to a full ranking of the alternatives. The argument $(criterion, r(a_1, a_2, \cdots, a_n))$ is created if the preference relations $a_1 \succ a_2 \succ \cdots \succ a_n$ exist in the set of agents preferences, i.e., the preference profile. For example, argument $(criterion, r(a_1, a_2, \cdots, a_n))$ means that in criterion $criterion$ there are agent(s) in which alternative a_1 is ranked above a_2, a_3 and so on until a_n. Also a_2 is ranked above a_3, a_4 and so on and all the possible preference relations that can appear in ranking $a_1, a_2, \cdots, a_n$. In other words, this ranking argument is extracted if there exist pairwise comparisons in all the agents in criterion $criterion$ that have pairs such that a_1 is above a_2, a_3, etc. Also, pairs such that a_2 is above a_3 are included and so on. If a "normal" ranking can not be defined in criterion $criterion$ from the preferences of the agents then argument $(criterion, blank)$ is defined which intuitively corresponds to a blank(undefined) vote. For example suppose that we have in a criterion the following pairwise comparisons which are given by all the agents: $a_1 \succ a_2, a_2 \succ a_3, a_1 \succ a_3, a_3 \succ a_2, a_2 \succ a_1$. Then an argument $(criterion, blank)$ is formed because there exist incompatible preference relations. Attacks are formed in the following cases:

   - Bidirectional attacks between preference relation arguments when in the same criterion different agents give different preference relation between two alternatives. For example, an attack $(criterion, r(a_i, a_j)) \leftrightarrow (criterion, r(a_j, a_i))$ is formed when in two agents a_i and a_j are ranked the opposite way.
   - Bidirectional attacks between ranking arguments, when we have different rankings in the same criterion. For example, an attack $(criterion, r1) \leftrightarrow (criterion, r2)$ is formed

when $r1$ and $r2$ are not the same.

- A single attack is formed between a preference relation argument and a ranking argument when the preference relation does not match the ranking. For example, an attack $(criterion, r(a_i, a_j)) \rightarrow (criterion, r1)$ is formed if in ranking $r1$, alternative a_j is ranked above a_i
- Bidirectional attacks between ranking arguments and the blank one(s), when they refer to the same criterion.

Then, we use the *preferred* semantics to compute the outcome of the framework and let for example, the result to be the following extensions $[1\_ca, 1\_bca, 1\_bc, 1\_blank]$. In the example, $1, 2$ are the criteria and $ca, bca, etc.$ are the rankings. The extensions correspond to virtual agents and for each one of them a grade for each alternative is computed by taking into account the criteria weight matrix. Hence, the virtual vote corresponding to the extension is a ranking of the alternatives according to their grade. In this way, we compute all the votes that constitute the "justified" preference profile. This profile is then reported to a voting rule (here, we use Tideman's) to compute the final collective winning ranking.

10. The agents report their preferences using the second method (ordinal). The arguments are of type $(criterion, rank)$ and 3 types of arguments are defined: preference relation, ranking, and blank argument(s). This method is similar to the abovementioned one but the difference is focused on the way the ranking arguments are formulated. In this case we take into account all the possible ranking arguments that can appear from the set of alternatives (i.e., the permutation without repetition on the set of alternatives). We take also into account rankings even if they do not appear in any agent. The attacks are formed as in the abovementioned method and we use the *preferred* semantics to compute the outcome of the framework and let for example, the result to be the following extensions $[1\_ca, 1\_bca, 1\_bc, 1\_blank]$. Here, it is also the case where $1, 2$ are the criteria and $ca, bca, etc.$ are the rankings. Also in this method the extensions correspond to virtual agents, for whom their ranking is computed from the grade of the alternatives (by taking into account the criteria weight matrix). Then, we compute all the votes that constitute the "justified" preference profile. This profile is then reported to a voting rule (here, we use Tideman's) to compute the final collective winning ranking.

Note that in methods 1-6 the second phase is done with argumentation techniques but it is easy to adapt the implemented algorithms and use instead voting methods to declare the winning ranking. Therefore, they constitute purely argumentative decision making procedures. The implementation of pure argumentative techniques stems from the need to identify the differences between them and combined (i.e., argumentation with voting) procedures. One should note that on the same input's profile different outcome can be produced by the various methods. The study of the extent of difference between the methods is let for future work.

Chapter 5

Conclusion - Future research

In the framework of this postdoctoral research, we focused on issues and problems that arise from social choice theory with an emphasis on preference aggregation for collective decision making. In large scale collective decision making, social choice is a normative study of how one ought to design a protocol for reaching consensus among the agents. However, in instances where the underlying decision problem is based on the group's preferences and the reasoning provided for them, standard voting methods used in social choice may be impractical for preference aggregation. That was our motivation: to design a mechanism which can seek for consensus among the group's agents, or at least a close to consensus group's collective decision. Our research hypothesis is that a decision made by a group of agents understanding the qualitative rationale behind each other's preferences has better chances to be as close as to a consensus decision. We proposed three mechanisms to handle and resolve inconsistencies among agents' preferences with the goal of finding the optimal collective solution (decision).

The first two methods followed an algorithmic approach which was based on the combination of argumentation framework and voting theory in order to propose solutions for the classical decision problem, while the third one was a method based on computational logic which applies to the multi-criteria decision problem. The first method was a novel qualitative decision process for which we showed that it can overcome some of the social choice deficiencies, such as the Condorcet paradox, and other important deficiencies regarding social choice properties. The second method was a quantitative one in which we proposed to quantify the deliberation phase by defining a new voting argumentation framework and its acceptability semantics. We proved for this method and its corresponding semantics some positive theoretical results by identifying under which conditions well-known properties that originate in argumentation and social choice theory can be satisfied.

The third method lies on resolving inconsistencies among agents' preferences when applied to the multi-criteria decision problem, an extension of the classical one. The difference between the classical decision problem and this one is that in the former the preferences are ordinal and no criteria are considered. The classical decision problem can be seen as equivalent to a single-criterion approach in the multi-criteria decision problem. Hence, the proposed method referred to solving the approach where agents give cardinal preferences over alternatives on different multiple criteria. We provided a novel modelling of this problem as an inconsistent knowledge base, and we explained how to benefit from the reasoning capabilities of the existential rules. The repairs of this knowledge base, which represent the maximally consistent point of views, and inference strategies can be used for providing a solution to collective decision making.

In the last phase of our research we turned our attention in real decision making applications using pure social choice techniques where we implemented methods and algorithms following a visualization approach. First, we designed a software application for decision making emphasizing on problems that can appear in agricultural engineering. The usage in agricultural engineering is is not exclusive because the application can be also used in other, more generic decision making problems. The tool uses methods of computational social choice and argumentation and consists of two main systems, i.e., the social choice system and the deliberation system. We mainly focused on the social choice system and demonstrated how it can be applied for agents' preference aggregation over different alternatives for valorization of materials used in agricultural engineering. We completed our research regarding the practical part by providing the implementation of all the algorithms used and needed to support the theoretical results of this research. The basic functionalities of these algorithms included the computation of the decision outcome which is build from argumentation frameworks and the computation, according to social choice methods, of the aggregation of the preference profile.

Our research has set many problems and inquiries that require further study and investigation. In the future, we envision the extension of our research in collective decision making when the reasoning is present by finding new techniques which are based in more complex argumentation frameworks. When we refer to more complex argumentation frameworks we mean to define new kind of voting frameworks that are not abstract or even define another kind of acceptability semantics and explore its properties. A nice idea would be to design semantics that permit us avoid the Condorcet paradox under any case or design semantics which are specially adapted to distinctive real decision problems. For example designing semantics with graded strength of generic arguments can be more appropriate for cases where long path of arguments refer to agents preferences. In addition, it will be very interesting to find also other reasoning methods that can model rational deliberation. On the applicative side, our goal is to design and implement the deliberation system which will complete the social choice system, and hence, the decision support software will be enriched with reasoning capabilities. Also, the implemented algorithms during this research can consist the core of future work regarding software tools of reasoning based methods on collective decision making and preference aggregation. Hence, these algorithms will be the foundational basis for building new software applications.

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