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Motion Coordination of Multiple Unicycle Robotic Vehicles under Operational Constraints in Obstacle-Cluttered Workspaces

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Abstract— **In this work, we consider the problem of coordinating the motion of a platoon of unicycle robots navigating inside an obstacle-cluttered workspace. We assume that each robot is equipped with on-board sensors that allow it to perceive nearby obstacles and obtain its relative position with respect to its preceding robot. Additionally, no robot other than the leader of the team is able to localize itself within the workspace and no centralized communication network exists, i.e., explicit information exchange between the agents is unavailable. To tackle this problem, we adopt a leader-follower architecture and propose a novel, decentralized control law for each robot-follower, based on the Prescribed Performance Control method, which guarantees collision-free tracking and visual connectivity maintenance by ensuring that each follower maintains its predecessor within its camera field of view while keeping static obstacles out of the line of sight for all times. Finally, we verify the efficacy of the proposed control scheme through simulations.**

I. Introduction

During the last decades, the cooperation of autonomous robotic platforms stands as an important research direction within the robotics field, owing to the multitude of advantages it entails, such as flexibility, increased capabilities and robustness. A particular class of robotic problems involves the coordination of the motion of multi-robot systems in order to achieve either a common or several independents goals. Centralized control schemes constitute a well studied solution to the coordination problem; nevertheless, high computational and communication costs render them viable only for teams with a small number of robots. Furthermore, owing to the complexity of the underlying problem, the scenarios dealt so far impose very strict assumptions, such as absence of static obstacles and knowledge of the state of the entire system, which cannot be met easily in real working conditions. Moreover, knowledge of the entire state by each robot necessitates the existence of a centralized localization system and communication network, which, apart from introducing issues such as time delays and inconsistencies, may not be available when considering unstructured workplaces.

On the other hand, decentralized control schemes for multi-robot systems bypass the aforementioned issues, offering a more efficient and robust solution to this problem [1], [2], although the design of decentralized control systems with guaranteed convergence and safety properties is, in general, a challenging task due to incongruities between the control objectives and the operational constraints. The authors in [3], [4] address the formation control problem of multi-agent systems by proposing a discrete switching technique to avoid collisions with obstacles in the workplace. In [5], the coordination of the multi-agent system subject to non-holonomic constraints was studied as a visual servoing problem. More specifically, the proposed solution was based on the design of an appropriate navigation function for the linearized dynamic model of the robots. Similar approaches are considered in [6]–[11]. However, all these methods require: i) sensors for each robot with a 360^o field of view, which have a relatively high cost and introduce a significant distortion to the visual data, and ii) that each follower knows the speed of its leader, which is rather unrealistic without assuming explicit information exchange between the robots.

On the other hand, the use of sensors with limited field of view imposes strict operational constraints, according to which the robot-follower should keep its predecessor in sight, to ensure the connectivity of the multi-robot system. An initial approach based on hybrid control techniques was presented in [12], [13] for a non-moving leader robot. Likewise, a method for computing optimal paths for a robot-follower that guarantee visual connectivity with a static robot-leader, have been proposed in [14]–[16]. Similar limitations were addressed in [17], [18] for a platoon of multiple vehicles. Finally, in [19], [20], the authors study the problem of maintaining visual connectivity from the context of game theory. Nonetheless, the aforementioned works do not consider environments occupied by obstacles, a strict assumption that reduces their applicability to real-world scenarios. Alternatively, the motion coordination problem of multi-robot systems, while ensuring visual connectivity in obstacle-cluttered workspaces, has been addressed with techniques based either on game theory [21]–[24] or on Artificial Potential Fields [25], [26]. Unfortunately, all these approaches follow the centralized control architecture, and thus, their efficiency drops as the size of the considered team increases.

In this work, we address the problem of coordinating the motion of a platoon of multiple unicycle robots, that operates within a workspace occupied by static obstacles. Each robot is equipped with proximity sensors that allow it to measure its distance with nearby obstacles and a forward looking camera with a limited field of view that allows it to detect and compute the position of its predecessor. Assuming that

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the robot that leads the platoon at the front traces a safe path inside the workplace, we propose a decentralized control law for the followers based on the Prescribed Performance Control (PPC) [27] method, which ensures safe navigation of the entire team using only local measurements from the aforementioned on-board sensors. Additionally, the proposed control scheme guarantees that the visual connectivity between neighbouring/successive robots is not compromised, i.e., each robot-follower maintains its predecessor within its camera field of view and prevents occlusions of the leader by the static obstacles. Finally, the main contributions of our work are summarized as follows:

- Contrary to the related literature on multi-robot coordination based on sensors with limited field of view, we propose a purely decentralized control protocol with guaranteed collision avoidance and visual connectivity maintenance.
- The proposed algorithm is easy to implement since it is of low complexity and does not require any explicit inter-robot information exchange via a communication network.
- Contrary to our previous related works [28], [29], the proposed scheme deals with generic workspaces involving static obstacles of irregular shape.

II. Problem Formulation

Let $\mathcal{W} \subset \mathbb{R}^2$ be a planar workspace occupied by n static obstacles $\mathcal{O}_i,\ i \in \mathfrak{I}_O$ with $\mathfrak{I}_O \triangleq \{1, 2, \ldots, n\}$, and let $\mathcal{W}_f \triangleq \mathcal{W} \setminus \cup_{i \in \mathfrak{I}_O} \mathcal{O}_i$ denote the free space. We consider a team of $N+1$ disk-shaped robots $\mathcal{R}_i$ of radius r_i, for $i \in \mathfrak{I}_R$ with $\mathfrak{I}_R \triangleq \{0, 1, \ldots, N\}$, which operate within $\mathcal{W}_f$ and whose motion obeys the unicycle kinematic model:

$$\begin{aligned} \dot{p}_i &= n_i \cdot u_i \\ \dot{\theta}_i &= \omega_i, \end{aligned} \tag{1}$$

where $p_i = [x_i, y_i]^T \in \mathbb{R}^2$ and $\theta_i \in \mathbb{R}$ denote the i-th robot's position and orientation w.r.t. an arbitrary inertial frame, respectively, $u_i,\ \omega_i \in \mathbb{R}$ denote the commanded linear and angular velocities, and $n_i = [\cos\theta_i, \sin\theta_i]^T \in \mathbb{R}^2$. We assume that all robots other than $\mathcal{R}_0$ are unable to either localize themselves within $\mathcal{W}_f$ or exchange explicitly information about their state with other robots. Consequently, they have to rely on on-board sensors for obtaining information about their environment and their neighbours. Particularly, each robot $\mathcal{R}_i \in \mathfrak{I}_F \triangleq \{1, 2, \ldots, N\}$ is equipped with a forward looking camera, fixed at its center, which acquires the relative position $\tilde{p}_i = p_{i-1} - p_i$ of robot $\mathcal{R}_{i-1}$ expressed in the camera's body-fixed frame, as long as robot $\mathcal{R}_{i-1}$ is visible by $\mathcal{R}_i$. Specifically, we say that robot $\mathcal{R}_{i-1}$ is visible by $\mathcal{R}_i$ if: i) $\mathcal{R}_{i-1}$ lies within the field of view $\mathcal{F}_i$ of the camera of robot $\mathcal{R}_i$, defined as a sector of angle $2\beta_{con} \in (0, \pi)$ and radius $d_{con} > 0$, and ii) the line segment $\mathcal{L}_i$ connecting $\mathcal{R}_{i-1}$ and $\mathcal{R}_i$ does not intersect any obstacle $\mathcal{O}_j,\ j \in \mathfrak{I}_O$ (see Figure 1). Moreover, $d_{col} > r_i + r_{i-1}$ denotes the minimum allowable distance between robots $\mathcal{R}_i$ and $\mathcal{R}_{i-1}$. Additionally, every robot $\mathcal{R}_i,\ i \in \mathfrak{I}_F$ is equipped with proximity sensors that enable it to perceive the unoccluded outline of nearby obstacles up to distance d_{con}, thus allowing it to compute the distances $d_{\mathcal{W}_{l,i}},\ d_{\mathcal{W}_{l,i}}$

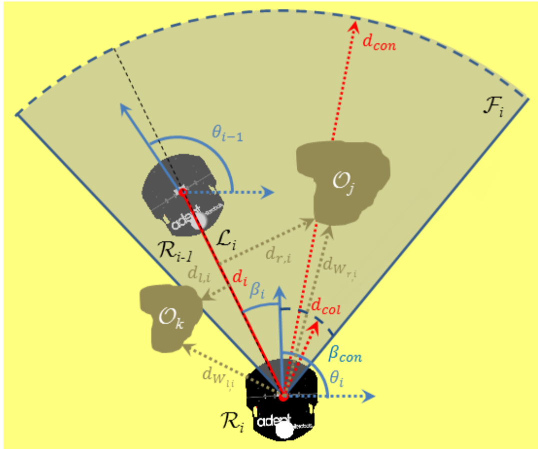

Fig. 1: Robot $\mathcal{R}_i$ tracking its predecessor $\mathcal{R}_{i-1}$.

between itself and the closest boundary of $\mathcal{W}_f$, as well as the distances $d_{l,i},\ d_{r,i}$ between the obstacles that are closest to the line of sight $\mathcal{L}_i$, from the left and right side respectively (see Figure 1).

Notice that the sensing capabilities of the robots described above define a line graph that is directed and rooted at robot $\mathcal{R}_0$. Moreover, let d_i and β_i be respectively the distance and angle of view corresponding to robot $\mathcal{R}_i$ and its predecessor $\mathcal{R}_{i-1}$, given by:

$$d_i = \|\tilde{p}_i\| \tag{2}$$

$$\beta_i = \operatorname{atan}\left(\frac{\tilde{y}_i}{\tilde{x}_i}\right) - \theta_i, \tag{3}$$

where $\tilde{p}_i = [\tilde{x}_i, \tilde{y}_i]^T$, for all $i \in \mathfrak{I}_F$. We now formally define the problem addressed in this work.

Problem 1: Given a feasible smooth path to be tracked by the leading robot $\mathcal{R}_0$ with bounded linear and angular velocity, design a decentralized control law for the robot velocities $u_i,\ \omega_i$, such that the entire team navigates safely within the workspace while avoiding inter-robot collisions and collisions with static obstacles, i.e.,

$$d_{col} < d_i(t),\ d_{\mathcal{W}_{l,i}}(t) > r_i,\ d_{\mathcal{W}_{r,i}}(t) > r_i \tag{4}$$

and every preceding robot $\mathcal{R}_{i-1}$ remains visible by its following robot $\mathcal{R}_i$, i.e.,

$$d_i(t) < d_{con},\ |\beta_i(t)| < \beta_{con},\ d_{l,i}(t) > 0,\ d_{r,i}(t) > 0 \tag{5}$$

for all $t > 0$ and $i \in \mathfrak{I}_F$. Additionally, whenever possible (owing to the aforementioned operational constraints) the formation should attain a desired inter-robot distance $d_\star \in (d_{col}, d_{con})$ with zero angle of view (i.e., each follower keeps its predecessor at the center of its camera field of view at distance $d_\star$).

Finally, to solve the aforementioned problem we assume that the path of the leading vehicle is smooth and feasible, in the sense that both the leading robot $\mathcal{R}_0$ as well as all followers $\mathcal{R}_i,\ i \in \mathcal{J}_F$ may track the safely the desired path without compromising the visibility constraints, and that the initial robot configuration satisfies:

$$d_{col} < d_i(0) < d_{con},\ |\beta_i(0)| < \beta_{con},$$
$$d_{\mathcal{W}_{l,i}}(0) > r_i,\ d_{\mathcal{W}_{r,i}}(0) > r_i,\ d_{r,i}(0) > 0$$

for all $i \in \mathfrak{I}_F$. It should be noted that the aforementioned suppositions are not strict since they establish the feasibility of the problem (i.e., there exists sufficient space for the each robot to track the desired path and keep its predecessor visible) and that initially all robots are safe and track their predecessors, such that the proposed control scheme may be applied[1].

III. Control Design

In this work, we employ the Prescribed Performance Control (PPC) design methodology [27] in order to meet the multiple safety specifications for collision avoidance and visibility maintenance, which are critical for the operation of the multi-robot team. Hence, let us first define the distance and angle of view errors:

$$e_{d_i}(t) = d_i(t) - d_\star,\ e_{\beta_i}(t) = \beta_i(t)$$

for each robot $\mathcal{R}_i$, $i \in \mathcal{J}_F$ with dynamics:

$$\dot{e}_{d_i} = -u_i \cos(\beta_i) + u_{i-1}\cos(\theta_i - \theta_{i-1} + \beta_i) \quad (6)$$
$$\dot{e}_{\beta_i} = -\omega_i + \frac{u_i}{d_i}\sin(\beta_i) - \frac{u_{i-1}}{d_i}\sin(\theta_i - \theta_{i-1} + \beta_i). \quad (7)$$

Based on the PPC approach, we shall design the velocity commands u_i, ω_i, such that:

$$\left.\begin{array}{l} \underline{\rho}_{d_i}(t) < e_{d_i}(t) < \overline{\rho}_{d_i}(t) \\ \underline{\rho}_{\beta_i}(t) < e_{\beta_i}(t) < \overline{\rho}_{\beta_i}(t) \end{array}\right\},\ \forall t \geq 0 \quad (8)$$

for appropriately selected performance functions $\underline{\rho}_{d_i}(t)$, $\overline{\rho}_{d_i}(t)$, $\underline{\rho}_{\beta_i}(t)$ and $\overline{\rho}_{\beta_i}(t)$ that should satisfy for all time the following properties:

$$\begin{array}{c} -(d_\star - d_{col}) < \underline{\rho}_{d_i}(t) < \overline{\rho}_{d_i}(t) < d_{con} - d_\star \\ -\beta_{con} < \underline{\rho}_{\beta_i}(t) < \overline{\rho}_{\beta_i}(t) < \beta_{con}. \end{array} \quad (9)$$

Notice that such design specifications for the distance and angle of view performance functions guarantee that each follower maintains the preceding robot within its camera field of view $\mathcal{F}_i$ and avoids collisions with it.

The aforementioned formulation was adopted successfully with exponential performance functions[2]:

$$\underline{\rho}_{d_i}(t) = -(d_\star - d_{col} - \rho_{d_\infty})e^{-\lambda t} - \rho_{d_\infty},$$
$$\overline{\rho}_{d_i}(t) = (d_{con} - d_\star - \rho_{d_\infty})e^{-\lambda t} + \rho_{d_\infty},$$
$$\underline{\rho}_{\beta_i}(t) = -(\beta_{con} - \rho_{\beta_\infty})e^{-\lambda t} - \rho_{\beta_\infty},$$
$$\overline{\rho}_{\beta_i}(t) = (\beta_{con} - \rho_{\beta_\infty})e^{-\lambda t} + \rho_{\beta_\infty}$$

[1] In case the robot-team is initially folded, which renders the aforementioned problem ill-defined, i.e., collision avoidance and visual connectivity cannot be met simultaneously, then an initial reordering of the line graph is needed to alleviate the deadlock. Notice that such reordering is compatible with our formulation since all following robots are considered identical with respect to sensing and actuation capabilities.

[2] The parameter λ dictates the exponential rate of convergence of the distance and angle of view errors e_{d_i}, e_{β_i} to small neighbourhoods of the origin with size ρ_{d_∞} and ρ_{β_∞} respectively. Notice that the properties in (9) are satisfied and hence the preceding robot is kept within the camera field of view of its follower.

in [28] for a platoon of multiple unicycle robots operating within an obstacle-free workspace. However, it should be noted that the presence of static obstacles within the workspace complicates significantly the problem at hand since: i) obstacles may break inter-robot visibility by raising occlusions among the robots if they stand between them, although the preceding vehicle may lie within the camera field of view of its follower, and ii) performing an obstacle avoidance manoeuvre may violate visual connectivity, thus compromising the safe operation of the multi-robot team. Therefore, in this work we propose to modify the control design presented in [28] by adapting the distance and angle of view performance functions $\underline{\rho}_{d_i}(t)$, $\overline{\rho}_{d_i}(t)$, $\underline{\rho}_{\beta_i}(t)$ and $\overline{\rho}_{\beta_i}(t)$ appropriately so that all operational specifications are met simultaneously.

In particular, when a single obstacle, either from the left or the right, depending on the motion of each pair of preceding-following robots, tends to intervene between them and raise either a visibility or collision risk, then we propose to deflect the angle of view β_i by modifying the corresponding performance functions $\underline{\rho}_{\beta_i}(t)$, $\overline{\rho}_{\beta_i}(t)$ (positively or negatively respectively, but still satisfying the field of view constraint β_{con}) so that the line of sight $\mathcal{L}_i$ moves away from the corresponding obstacle, thus ensuring that neither a collision nor a visibility break occurs. However, during the aforementioned maneuver to avoid an obstacle either from the left or the right, another obstacle at the opposite side may intervene, thus introducing a conflict, since the deviation of the angle of view is not sufficient to bypass the obstacles owing to their contradicting effects on the control algorithm (i.e., the obstacle at the left of the line of sight will lead the angle of view to positive values whereas the obstacle at the right to negative values). Fortunately, in such critical case the solution to the follower's control problem is to approach its preceding robot by reducing the distance performance functions $\underline{\rho}_{d_i}(t)$, $\overline{\rho}_{d_i}(t)$ but keeping the inter-robot distance greater than d_{col} to avoid collision. Notice that the aforementioned strategy is viable since we have assumed that the path of the leading robot $\mathcal{R}_0$ is safe for the whole robot-team under the considered operational specifications.

Based on the aforementioned formulation, we design the following update laws for the distance and angle of view performance functions:

$$\dot{\underline{\rho}}_{d_i} = -\lambda\left(\underline{\rho}_{d_i} + \rho_{d_\infty}\right) - F_{d,i}(F_{r,i} + F_{l,i}),$$
$$\text{with } \underline{\rho}_{d_i}(0) = -(d_\star - d_{col}) \quad \text{(10a)}$$
$$\dot{\overline{\rho}}_{d_i} = -\lambda\left(\overline{\rho}_{d_i} - \rho_{d_\infty}\right) - F_{d,i}(F_{r,i} + F_{l,i}),$$
$$\text{with } \overline{\rho}_{d_i}(0) = d_{con} - d_\star \quad \text{(10b)}$$
$$\dot{\underline{\rho}}_{\beta_i} = -\lambda\left(\underline{\rho}_{\beta_i} + \rho_{\beta_\infty}\right) + F_{r,i} - F_{l,i},$$
$$\text{with } \underline{\rho}_{\beta_i}(0) = -\beta_{con} \quad \text{(10c)}$$
$$\dot{\overline{\rho}}_{\beta_i} = -\lambda\left(\overline{\rho}_{\beta_i} - \rho_{\beta_\infty}\right) + F_{r,i} - F_{l,i},$$
$$\text{with } \overline{\rho}_{\beta_i}(0) = \beta_{con} \quad \text{(10d)}$$

where $F_{l,i} \triangleq \frac{\boldsymbol{S}\left(\min\left\{d_{l,i}, d_{W_l,i} - r_i\right\}, \delta\right)}{\min\left\{d_{l,i}, d_{W_l,i} - r_i\right\}}$, $F_{r,i} \triangleq$

$\frac{\boldsymbol{S}(\min\{d_{r,i}, d_{W_r,i}-r_i\},\delta)}{\min\{d_{r,i}, d_{W_r,i}-r_i\}}$ and $F_{d,i} \triangleq \boldsymbol{S}\left(|F_{r,i}-F_{l,i}|,\delta\right)$, with $\boldsymbol{S}(\star,\delta) = 1-\frac{\star}{\delta}$, $\forall\star\in[0,\delta)$ and $\boldsymbol{S}(\star,\delta)=0$, $\forall\star\in[\delta,\infty)$, for a positive constant δ. Notice that when the distance of the robot $\mathcal{R}_i$ and the line segment $\mathcal{L}_i$ with the surrounding obstacles is large ($>\delta$) then both terms $F_{r,i}$ and $F_{l,i}$ vanish and consequently the aforementioned update laws yield exponential response similar to [28]. On the other hand, when a single obstacle intervenes from the left or the right between a follower and its predecessor then the term $F_{l,i}$ or $F_{r,i}$ respectively increases, causing the angle of view performance functions to decrease or increase and consequently the robot and the line segment $\mathcal{L}_i$ to move away from the obstacle. However, when obstacles are close to the robot or the line segment $\mathcal{L}_i$ from both sides, then $F_{r,i}$ and $F_{l,i}$ increase almost equally, thus activating $F_{d,i}$ and causing the distance performance functions to decrease, so that the following robot approaches its predecessor travelling in between the obstacles. Finally, in order to ensure that the desired properties (9) of the distance and angle of view performance functions $\underline{\rho}_{d_i}(t)$, $\overline{\rho}_{d_i}(t)$, $\underline{\rho}_{\beta_i}(t)$ and $\overline{\rho}_{\beta_i}(t)$ are met for all time, we also apply a standard Lipschitz continuous projection operator [30] on the aforementioned update laws over the sets: $[-(d_\star-d_{col}), d_{con}-d_\star-2\rho_{d_\infty}]$, $[-(d_\star-d_{col})-2\rho_{d_\infty}, d_{con}-d_\star]$, $[-\beta_{con},\beta_{con}-2\rho_{\beta_\infty}]$ and $[-\beta_{con}+2\rho_{\beta_\infty},\beta_{con}]$, respectively.

Subsequently, we present the velocity control protocol for each robot $\mathcal{R}_i$, $i\in\mathcal{J}_F$ that establishes prescribed performance with respect to the aforementioned performance functions (10) by guaranteeing the inequalities (8) for the distance and angle of view errors for all time. More specifically, we first define the transformed errors $\varepsilon_{d_i}\triangleq\ln\left(\frac{e_{d_i}-\overline{\rho}_{d_i}}{\overline{\rho}_{d_i}-e_{d_i}}\right)$ and $\varepsilon_{\beta_i}\triangleq\ln\left(\frac{e_{\beta_i}-\overline{\rho}_{\beta_i}}{\overline{\rho}_{\beta_i}-e_{\beta_i}}\right)$. Notice that if we manage to keep the transformed error signals $\varepsilon_{d_i}(t)$ and $\varepsilon_{\beta_i}(t)$ bounded for all time[3], via the appropriate selection of the velocity commands, then it is easy to check that we also guarantee (8) for all time, no matter how large the upper bound of $\varepsilon_{d_i}(t)$ and $\varepsilon_{\beta_i}(t)$ is. Consequently, the problem at hand, as described by (8), has been recast as a simple stabilization problem of the transformed error signals $\varepsilon_{d_i}(t)$ and $\varepsilon_{\beta_i}(t)$, which can be resolved by the following velocity control protocol:

$$u_i = \frac{1}{\cos(\beta_i)}\left(k_d\varepsilon_{d_i} - \frac{\dot{\underline{\rho}}_{d_i}\left(\overline{\rho}_{d_i}-e_{d_i}\right)+\dot{\overline{\rho}}_{d_i}\left(e_{d_i}-\underline{\rho}_{d_i}\right)}{\overline{\rho}_{d_i}-\underline{\rho}_{d_i}}\right) \tag{11}$$

$$\omega_i = \frac{u_i}{d_i}\sin(\beta_i)+k_\beta\varepsilon_{\beta_i} - \frac{\dot{\underline{\rho}}_{\beta_i}\left(\overline{\rho}_{\beta_i}-e_{\beta_i}\right)+\dot{\overline{\rho}}_{\beta_i}\left(e_{\beta_i}-\underline{\rho}_{\beta_i}\right)}{\overline{\rho}_{\beta_i}-\underline{\rho}_{\beta_i}}. \tag{12}$$

with positive control gains k_d and k_β.

Theorem 1: Consider a team of unicycle robots that operates within a planar and obstacle cluttered environment, under the safety and visibility constraints that were described in Section II. Moreover, assume that the leading robot of the team $\mathcal{R}_0$ follows a safe and feasible path within the workspace and that initially at $t=0$ all safety and visibility constraints are satisfied. The proposed decentralized control protocol (11) and (12) along with the update laws that modify the performance functions (10) navigates safely the robot team within the workspace by avoiding any collisions and visibility breaks.

Proof: Based on the formulated problem the underlying graph of the multi-robot team comprises a directed line graph rooted to the leading vehicle $\mathcal{R}_0$. Therefore, the analysis may be broken down into pairs of preceding and following robots starting from the leading one until the last. Thus, let us define the positive definite function of the transformed errors:

$$V_i = \tfrac{1}{2}\varepsilon_{d_i}^2 + \tfrac{1}{2}\varepsilon_{\beta_i}^2$$

Differentiating with respect to time and invoking the error dynamics (6) and (7), we get:

$$\begin{aligned}\dot{V}_i =& \frac{\varepsilon_{d_i}\left(\overline{\rho}_{d_i}-\underline{\rho}_{d_i}\right)}{\left(\overline{\rho}_{d_i}-e_{d_i}\right)\left(e_{d_i}-\underline{\rho}_{d_i}\right)}\Bigg(u_{i-1}\cos(\theta_i-\theta_{i-1}+\beta_i)\\ &- u_i\cos(\beta_i) - \frac{\dot{\underline{\rho}}_{d_i}\left(\overline{\rho}_{d_i}-e_{d_i}\right)+\dot{\overline{\rho}}_{d_i}\left(e_{d_i}-\underline{\rho}_{d_i}\right)}{\overline{\rho}_{d_i}-\underline{\rho}_{d_i}}\Bigg)\\ &+\frac{\varepsilon_{\beta_i}\left(\overline{\rho}_{\beta_i}-\underline{\rho}_{\beta_i}\right)}{\left(\overline{\rho}_{\beta_i}-e_{\beta_i}\right)\left(e_{\beta_i}-\underline{\rho}_{\beta_i}\right)}\Bigg(-\frac{u_{i-1}}{d_i}\sin(\theta_i-\theta_{i-1}+\beta_i)\\ &-\omega_i+\frac{u_i}{d_i}\sin(\beta_i) - \frac{\dot{\underline{\rho}}_{\beta_i}\left(\overline{\rho}_{\beta_i}-e_{\beta_i}\right)+\dot{\overline{\rho}}_{\beta_i}\left(e_{\beta_i}-\underline{\rho}_{\beta_i}\right)}{\overline{\rho}_{\beta_i}-\underline{\rho}_{\beta_i}}\Bigg).\end{aligned}$$

Hence, substituting the proposed control protocol (11) and (12), we arrive at:

$$\begin{aligned}\dot{V}_i =& \frac{\varepsilon_{d_i}\left(\overline{\rho}_{d_i}-\underline{\rho}_{d_i}\right)}{\left(\overline{\rho}_{d_i}-e_{d_i}\right)\left(e_{d_i}-\underline{\rho}_{d_i}\right)}\left(u_{i-1}\cos(\theta_i-\theta_{i-1}+\beta_i)-k_d\varepsilon_{d_i}\right)\\ &-\frac{\varepsilon_{\beta_i}\left(\overline{\rho}_{\beta_i}-\underline{\rho}_{\beta_i}\right)}{\left(\overline{\rho}_{\beta_i}-e_{\beta_i}\right)\left(e_{\beta_i}-\underline{\rho}_{\beta_i}\right)}\left(k_\beta\varepsilon_{\beta_i}+\frac{u_{i-1}}{d_i}\sin(\theta_i-\theta_{i-1}+\beta_i)\right)\end{aligned}$$

Notice also that by design (owing to the projection applied on (10)) the distance and angle of view performance functions guarantee that if the corresponding errors evolve within them as dictated by (8) then $d_i > d_{col} > 0$ and $|\beta_i| < \beta_{con} < \frac{\pi}{2}$. Moreover, the terms $\frac{\overline{\rho}_{d_i}-\underline{\rho}_{d_i}}{\left(\overline{\rho}_{d_i}-e_{d_i}\right)\left(e_{d_i}-\underline{\rho}_{d_i}\right)}$ and $\frac{\overline{\rho}_{\beta_i}-\underline{\rho}_{\beta_i}}{\left(\overline{\rho}_{\beta_i}-e_{\beta_i}\right)\left(e_{\beta_i}-\underline{\rho}_{\beta_i}\right)}$ are strictly positive. Finally, the velocity u_{i-1} is bounded by induction starting from the velocity of the leading robot $\mathcal{R}_0$. Consequently, the right terms in both parenthesis are bounded and we easily deduce that the transformed errors ε_{d_i} and ε_{β_i} are uniformly ultimately bounded. As a result, the prescribed performance encapsulated by the inequalities (8) is satisfied for all time and thus neither collisions nor visibility breaks occur. Furthermore, all closed loop system signals remain bounded, which completes the proof. ■

[3]Owing to the appropriately selected initial value of the performance functions (10) and the assumption that the robot configuration meets initially all the operational specifications, the transformed errors are finite at $t=0$.

Remark 1: It should be noted that the proposed control protocol (11) and (12) along with the update laws (10) employs information that is exclusively acquired by the forward looking camera and the proximity sensors that are mounted on each robot. Thus, its implementation is purely decentralized and, contrary to other works in the related literature, does not necessitate for any explicit network communication among the robots, e.g., communicating information for the velocity of the preceding robot. Moreover, notice that the operational specifications are satisfied via the appropriate modification of the performance functions (10), hence simplifying the selection of the control gains k_d and k_β. Nevertheless, it should be stressed that their values affect both the response of the distance and angle of view errors within the corresponding performance bounds as well as the control signal. Therefore, additional fine tuning might be needed in real robot implementation to meet the actuation constraints.

IV. Simulation Results

To validate the aforementioned control protocol, we have conducted an extensive simulation study in MATLAB for a team of 7 following robots and a leading one, operating within a complex workspace that involves narrow passages through which the leading robot safely navigates (see Fig. 2). The radius of the robots is $r_i = 0.2\text{m}$, $i \in \mathcal{J}_F$, the desired inter-robot distance is $d_\star = 2\text{m}$ and the operational constraints are set as $d_{col} = 0.5\text{m}$, $d_{con} = 4\text{m}$ and $\beta_{con} = \frac{45\pi}{180}\text{rad}$. Moreover, we selected the following performance function parameters $\lambda = 1$, $\rho_{d_\infty} = 0.1\text{m}$, $\rho_{\beta_\infty} = 0.1\text{rad}$. Finally, the parameters of the control protocol were chosen as $k_d = k_\beta = 4$ and $\delta = 0.5$.

The results are given in Figs. 2 and 3. More specifically, Fig. 2 depicts 8 successive snapshots of the robot team within the workspace, every 10 seconds, along with the camera field of view of each following robot. Notice that initially at $t = 0\text{sec}$ all preceding robots lie within the camera field of view of their followers and are kept within it for all time after, despite the sharp corners of the considered narrow workspace. Additionally, the evolution of the distance and angle of view errors is given in Fig. 3, along with the corresponding performance functions and the operational specifications. Apparently, the proposed decentralized control protocol retained the distance and angle of view errors within the performance envelope without compromising the the safety of the multi-robot team (i.e., neither collisions nor visibility breaks occurred). Finally, the operation of the multi-robot teams is demonstrated by the video at the following link:

https://youtu.be/yRBteQSzeVQ

V. Conclusions

In this work, we tackled the problem of coordinating the motion of a platoon of unicycle robots that navigate within an obstacle-cluttered workspace. Given that each robot is equipped with proximity sensors for detecting nearby obstacles and a forward looking camera for tracking the preceding robot, we developed a safe decentralized control strategy that avoids collisions while maintaining visual connectivity between every pair of successive robots for all time. Finally, simulations results were presented that demonstrate the efficacy of the proposed control scheme.

Future research efforts will be devoted towards incorporating hard constraints on the velocity commands to increase the applicability of our approach. Moreover, we plan to verify the theoretical findings via experimental results, employing real unicycle robotic vehicles.

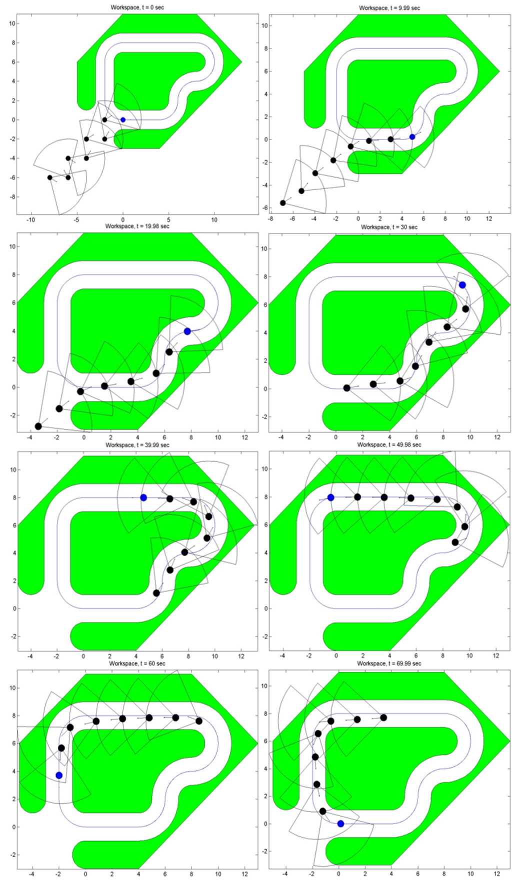

Fig. 2: The operation of the robot-team (blue color indicates the leading robot and black the following ones) is depicted for 8 consecutive time instants, every 10 seconds. Each camera field of view is given by the black quadrants.

References

[1] A. Jadbabaie, J. Lin, and A. S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Transactions on Automatic Control, vol. 48, no. 6, pp. 988-1001, 2003.

[2] J. A. Fax and R. M. Murray, Information flow and cooperative control of vehicle formations, IEEE Transactions on Automatic Control, vol. 49, no. 9, pp. 1465-1476, 2004.

[3] J. P. Desai, J. P. Ostrowski, and V. Kumar, Modeling and control of formations of nonholonomic mobile robots, IEEE Transactions on Robotics and Automation, vol. 17, no. 6, pp. 905-908, 2001.

[4] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, A vision-based formation control framework, in Proc. of the

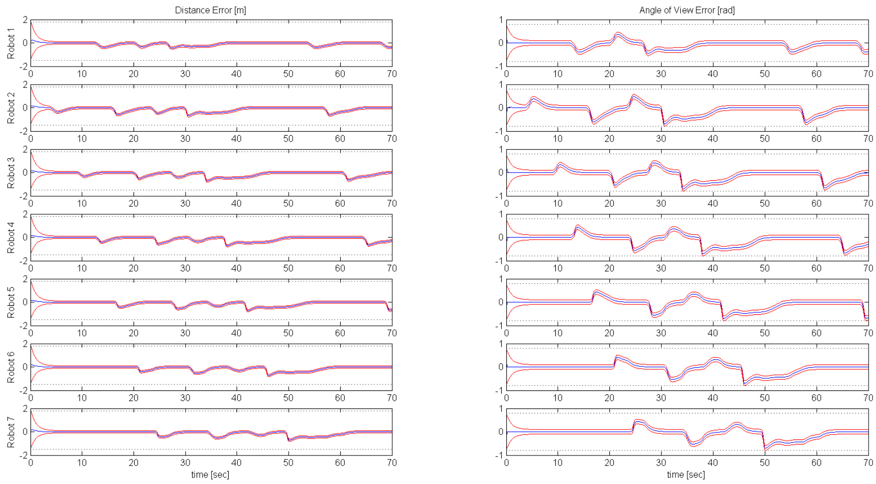

Fig. 3: The evolution of the distance and angle of view errors (blue solid lines) along with the corresponding performance functions (red solid lines) and safety constraints (black dashed lines).

IEEE Transactions on Robotics and Automation, vol. 18, no. 5, pp. 813-825, 2002.

[5] N. Cowan, O. Shakernia, R. Vidal, and S. Sastry, Vision-based follow the leader, in Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 1796-1801, 2003.

[6] G. L. Mariottini, F. Morbidi, D. Prattichizzo, G. J. Pappas, and K. Daniilidis, Leader-follower formations: Uncalibrated vision-based localization and control, in Proc. of the IEEE International Conference on Robotics and Automation, pp. 2403-2408, 2007.

[7] G. L. Mariottini, F. Morbidi, D. Prattichizzo, N. Vander Valk, G. Pappas, and K. Daniilidis, Vision-based localization for leader- follower formation control, IEEE Transactions on Robotics, vol. 25, no. 6, pp. 1431-1438, 2009.

[8] O. A. Orqueda and R. Fierro, Robust vision-based nonlinear formation control, in Proc. of the IEEE American Control Conference, pp. 1422-1427, 2006.

[9] H. Kannan, V. K. Chitrakaran, D. M. Dawson, and T. Burg, Vision based leader/follower tracking for nonholonomic mobile robots, in Proc. of the IEEE American Control Conference, pp. 2159-2164, 2007.

[10] X. Liang, Y. Liu, H. Wang, W. Chen, K. Xing and T. Liu, Leader-Following Formation Tracking Control of Mobile Robots Without Direct Position Measurements, IEEE Transactions on Automatic Control, vol. 61, no. 12, pp. 4131-4137, 2016.

[11] H. J. Min, A. Drenner, and N. Papanikolopoulos, Vision-based leader follower formations with limited information, in Proc. of the IEEE International Conference on Robotics and Automation, pp. 351-356, 2009.

[12] G. Kantor and A. A. Rizzi, Feedback control of underactuated systems via sequential composition: Visually guided control of a unicycle, in Robotics Research, P. Dario and R. Chatila, Eds. Springer Berlin Heidelberg, pp. 281290, 2005.

[13] G. A. D. Lopes and D. E. Koditschek, Visual servoing for nonholonomically constrained three degree of freedom kinematic systems, International Journal of Robotics Research, vol. 26, no. 7, pp. 715-736, 2007.

[14] S. Bhattacharya, R. Murrieta-Cid, and S. Hutchinson, Optimal paths for landmark-based navigation by differential-drive vehicles with field of- view constraints, IEEE Transactions on Robotics, vol. 23, no. 1, pp. 47-59, 2007.

[15] P. Salaris, D. Fontanelli, L. Pallottino, and A. Bicchi, Shortest paths for a robot with nonholonomic and field-of view constraints, IEEE Transactions on Robotics, vol. 26, no. 2, pp. 269-280, 2010.

[16] S. Maniatopoulos, D. Panagou, K. J Kyriakopoulos, Model predictive control for the navigation of a nonholonomic vehicle with field- of-view constraints, in Proc. of the IEEE American Control Conference, pp. 3967-3972, 2013.

[17] F. Morbidi, F. Bullo, and D. Prattichizzo, Visibility maintenance via controlled invariance for leader-follower vehicle formations, Automatica, vol. 47, no. 5, pp. 1060-1067, 2011.

[18] X. Liang, H. Wang, Y. Liu, W. Chen and T. Liu, Formation Control of Nonholonomic Mobile Robots Without Position and Velocity Measurements, IEEE Transactions on Robotics, vol. 34, no. 2, pp. 434-446, 2018.

[19] R. Murrieta-Cid, U. Ruiz, J. L. Marroquin, J.-P. Laumond, and S. Hutchinson, Tracking an omnidirectional evader with a differential drive robot, Autonomous Robots, vol. 18, no. 4, pp. 345-366, 2011.

[20] U. Ruiz and R. Murrieta-Cid, A homicidal differential drive robot, in Proc. of the IEEE International Conference on Robotics and Automation, pp. 3218-3225, 2012.

[21] S. M. LaValle, H. H. Gonzalez-Banos, C. Becker, and J.-C. Latombe, Motion strategies for maintaining visibility of a moving target, in Proc. of the IEEE International Conference on Robotics and Automation, pp. 731-736, 1997.

[22] R. Murrieta-Cid, L. Munoz-Gomez, M. Alencastre-Miranda, A. Sarmiento, S. Kloder, S. Hutchinson, F. Lamiraux, and J. P. Laumond, Maintaining visibility of a moving holonomic target at a fixed distance with a non-holonomic robot, in Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 2687-2693, 2005.

[23] R. Murrieta-Cid, R. Monroy, S. Hutchinson, and J.-P. Laumond, A complexity result for the pursuit-evasion game of maintaining visibility of a moving evader, in Proc. of the IEEE International Conference on Robotics and Automation, pp. 2657-2664, 2008.

[24] R. Murrieta-Cid, T. Muppirala, A. Sarmiento, S. Bhattacharya, and S. Hutchinson, Surveillance strategies for a pursuer with finite sensor range, International Journal of Robotics Research, vol. 26, no. 3, pp. 233-253, 2007.

[25] D. Panagou, and V. Kumar, Cooperative visibility maintenance for leader-follower formations in obstacle environments, IEEE Transactions on Robotics, vol. 30, no. 4, pp. 831-844, 2014.

[26] D. Sakai, H. Fukushima and F. Matsuno, Leader-Follower Navigation in Obstacle Environments while Preserving Connectivity Without Data Transmission, IEEE Transactions on Control Systems Technology, vol. 26, no. 4, pp. 1233-1248, 2018.

[27] C. P. Bechlioulis and G. A. Rovithakis, Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance, IEEE Transactions on Automatic Control, vol. 53, no. 9, pp. 2090-2099, 2008.

[28] C. K. Verginis, C. P. Bechlioulis, D. V. Dimarogonas and K. J. Kyriakopoulos, Decentralized 2-D control of vehicular platoons under limited visual feedback, in Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 3566-3570, 2015.

[29] I. Delimpaltadakis, C. P. Bechlioulis and K. J. Kyriakopoulos, Decentralized Platooning with Obstacle Avoidance for Car-like Vehicles with Limited Sensing, IEEE Robotics and Automation Letters, vol. 3, no.2, pp. 835-840, 2018.

[30] Z. Cai, M. de Queiroz, and D. Dawson, A sufficiently smooth projection operator, IEEE Transactions on Automatic Control, vol. 51, no. 1, pp. 135139, 2006.