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Earthquake Networks as a Tool for Seismicity Investigation: a Review

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Abstract—Seismic hazard assessment is one of the main targets of seismological research, aiming to contribute to reducing the catastrophic consequences of strong earthquakes (e.g., $M \geq 6.0$). From the early stage of seismological research, both purely seismological and statistical methods were adopted for seismic hazard assessment. An approach towards this target was attempted by means of network theory, aiming to provide insight into the complex physical mechanisms that cause earthquakes and whether the occurrence of strong earthquakes can be predicted to some extent. Application of network theory in different areas of the world with intense seismic activity, such as Japan, California, Italy, Greece, Iran, and Chile, has yielded promising results that have negligible probability of being obtained by purely random guessing.

Key words: Nodes, Connections, Network measures, Randomized networks, Correlation, Time series, Main shocks.

1. Introduction

Investigation of seismicity properties using statistical tools has attracted great interest from Earth scientists due to its potential to reveal significant components of complex seismogenic processes. The first empirical statistical approaches were the Omori (1894) and Gutenberg–Richter (1944) laws, which allowed analysis and understanding of the earthquake time and magnitude distribution. The Omori law expresses the decay rate of aftershocks after a main shock, whereas the Gutenberg–Richter law states that the earthquake occurrence frequency follows a power law with respect to the energy released. Since then, various statistical models that have made a particular contribution to seismic hazard assessment have been proposed, such as the Poisson (Cornell 1968; Lomnitz 1974), Markov (Nava et al. 2005; Herrera et al. 2006; Votsi et al. 2013), semi-Markov (Altınok 1991; Altınok and Kolçak 1999; Votsi et al. 2012, 2014), and regional earthquake likelihood models (RELM) (Kagan and Jackson 1994; Helmstetter et al. 2007; Holliday et al. 2007; Rhoades 2007), among others.

Investigation of the complex seismicity behavior constitutes a major scientific challenge and an indispensable component for improving knowledge concerning seismogenesis and earthquake forecasting. For statistical analysis to be efficient in revealing certain nonlinear spatiotemporal structures, continuously more advanced tools must be applied. One approach for investigating the spatial and temporal complexity of seismicity is via the construction of earthquake networks, which have been proved to represent a powerful tool to provide information on the topology and dynamics of complex systems. Each graph or network is defined by its nodes and the connections among them. The nodes of the network are usually assumed to represent distinct subsystems, and the connections the interactions among them. The pioneer of graph theory was Leonhard Euler, who published in 1735 the solution to the Königsberg bridge problem, where an itinerant merchant must traverse each of the bridges of the Prussian city of Königsberg just once. In recent years, network theory was been successfully applied in various disciplines, such as economics (Emmert-Streib and Dehmer 2010; Billio et al. 2012; Heiberger 2014; Fiedor 2014; Papana et al. 2017), biology (Jeong et al. 2001; Girvan and Newman 2002; Wang and Chen 2003), climatology (Donges et al. 2009; Bialonski et al. 2010; Palus et al. 2011), meteorology (Hlinka et al. 2012), neuroscience (Rubinov and Sporns 2010; Kugiumtzis and Kimiskidis 2015; Bullmore et al. 2016; Kugiumtzis et al. 2017), the spread of forest fires (Belkacem et al. 2015), physiology (Porta and

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Faes 2016), transportation (Wang et al. 2017; Aydin et al. 2017), and the spread of viruses (Zhang and Gan 2018).

Complex network analysis was introduced in seismology by Abe and Suzuki (2004a) to study seismicity as a spatiotemporal complex system, providing greater insight into and perspective on seismicity patterns. The global physical properties of seismicity can be explored by examining the geometrical (topological) and dynamic characteristics of earthquake networks. Interesting nontrivial properties that indicate their specific structures have been revealed, such as the small-world (Watts and Strogatz 1998) and scale-free (Barabasi and Albert 1999) properties, which indicate the underlying organization principles of earthquake networks. Changes in the structure of the network, as determined by the evolution of network measures, can reveal precursory seismicity patterns a few days before the occurrence of a main shock. Therefore, we focus on applications based on network measures and nontrivial properties of networks for seismicity investigation as a complex system.

Section 2 presents the construction of networks along with the network measures, whereas Sect. 3 describes some network theory applications for seismic hazard assessment. Concluding remarks are presented in Sect. 4.

2. *Methodology of Earthquake Networks*

The construction of an earthquake network and its measures along with approaches for random network construction are presented in this section. The following tools are used to investigate nontrivial properties and the evolution of network measures:

2.1. *Building the Earthquake Network*

An earthquake network is defined by a graph $G = (K, E)$, where K denotes the set of nodes and E the set of connections among them. The nodes in seismicity studies are created in two different ways, with the study area being superposed by a normal grid (Fig. 1) of two-dimensional (2-D) seismic cells (Abe and Suzuki 2004a; Baiesi and Paczuski 2004) or

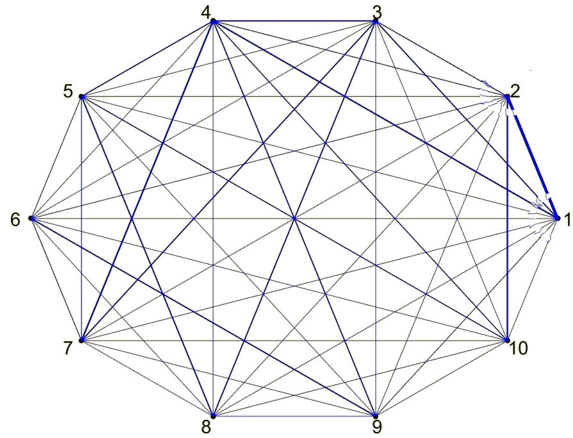

Figure 1
Typical approach for construction of earthquake network by Abe and Suzuki (2004a)

being divided into K subareas that constitute seismic zones defined on the basis of the homogeneity of their seismotectonic properties (among which the faulting type and seismic moment rate are the most important) (Chorozoglou et al. 2017). The connections in the earthquake network are defined either by successive earthquakes (Abe and Suzuki 2004a; Baiesi and Paczuski 2004) or by the significant correlation between the seismic activity in two seismic zones or cells, when time series are constructed and used (Jimenez et al. 2008; Tenenbaum et al. 2012; Chorozoglou et al. 2017).

The construction of earthquake networks, when the connections among the nodes are given by successive earthquakes, can be based on the approaches of Abe and Suzuki (2004a) or Baiesi and Paczuski (2004). In both cases, the study area is divided into cells that are considered as the nodes of the earthquake networks, inside which the earthquakes occur, while the connections are given either by the succession of earthquakes (Abe and Suzuki 2004a) or by the unifying scaling relation (Bak et al. 2002) between the occurrence time of successive earthquakes, the spatial distance of their epicenters, and their magnitudes, as described by Baiesi and Paczuski (2004). The construction of an earthquake network according to the approach of Abe and Suzuki (2004a) is based on a division of the study area into numerous small cubic cells. A cell is regarded as a node if earthquakes with any values of magnitude occurred therein. Two successive events define a connection between two nodes. If two successive events occur in the same cell, they form a loop. This procedure enables mapping of the seismic data to a growing random graph. The construction of an earthquake network based on the approach of Baiesi and Paczuski (2004) proposes a unified scaling law for the waiting times between earthquakes, expressing a hierarchical organization in time, space, and magnitude. The waiting time interval for the crossover between two regimes for earthquakes larger than a given magnitude depends on the area and magnitude under consideration. There is a linear regime, indicating a power-law distribution, extending up to a cutoff, indicating an upper limit on the waiting time. For fixed cell size and increasing cutoff, the range of the power-law regime increases. For fixed cutoff and increasing cell size, the range of the power-law regime decreases. It has been proven that the approach of Abe and Suzuki (2004a) exhibits universal behavior (Abe and Suzuki 2012), whereas the approach of Baiesi and Paczuski (2004) depends on the datasets used for the network construction (Carbone et al. 2005).

The connections and loops represent the correlations between two successive earthquakes. For simple connections, the adjacency matrix $A = \{a_{ij}\}, i,j \in \{1,\ldots,K\}$ acquires components assigned the value of 1 when earthquakes occur successively, and 0 otherwise. In this way, a seismic region or cell can be correlated with another one far away, which is consistent with observations of "remotely triggered seismicity" (Hill et al. 1993). Such an interpretation is also confirmed by the hypothesis that seismicity is a self-organized phenomenon (Bak and Tang 1989; Wanliss et al. 2017). It has been shown that an earthquake may be triggered by an immediately preceding strong one located at a distance of more than 1000 km (Steeples and Steeples 1996). The weight matrix $W = \{w_{ij}\}, i,j \in \{1,\ldots,K\}$ is constructed as follows: A square nonsymmetric matrix $S = \{s_{ij}\}$ is created, where s_{ij} takes either an integer value, indicating how many times the succession of earthquakes appears for each pair (i,j) of nodes, or a positive real value, indicating the cumulative seismic moment, ΣM_0, released in all successions of earthquakes between nodes i and j, where the seismic moment, M_0, of an earthquake of magnitude M is given by $\log M_0 = 1.5M + 16.01$ (Kanamori and Anderson 1975). For both cases, the components s_{ij} are normalized by their maximum to give the weights $w_{ij} = s_{ij}/\max\{s_{ij}\}$ in the matrix W (Chorozoglou et al. 2017).

When time series are constructed and used, the observed variables are either the cumulative seismic moment, ΣM_0 (Tenenbaum et al. 2012), or the number of earthquakes (Jimenez et al. 2008) within each seismic cell or zone. The connections are given by the linear zero-lag cross-correlation, which is actually Pearson's correlation coefficient. Thus, the connections are not introduced based on successive earthquakes, as various studies have shown that the distribution of interevent times is not Poissonian (Omori 1894; Corral 2004; Livina et al. 2005; Lippiello et al. 2008; Lennartz et al. 2008). Hence, successive earthquakes may not be the result of uncorrelated independent probability, but dependent on the long-term history for each seismic area (node). Then, a set is considered of K random variables observed at n time points $\{X_{1,t},\ldots,X_{K,t}\}$, for $t = 1,\ldots,n$, where each variable is the cumulative seismic moment, ΣM_0, or the number of earthquakes inside each node area. For two variables $X = X_i$ and $Y = Y_j$, $i,j \in \{1,..,K\}$, the Pearson's correlation coefficient is defined as

$$r_{X,Y} = \frac{S_{XY}}{\sqrt{S_X^2 S_Y^2}}, \quad (1)$$

where $S_{XY} = \frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})(y_t - \bar{y})$ is the sample covariance of (X,Y), $S_X^2 = \frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})^2$ and $S_Y^2 = \frac{1}{n-1}\sum_{t=1}^{n}(y_t - \bar{y})^2$ are the sample variances of X and Y, and $\bar{x}$ and $\bar{y}$ are the sample means of X and Y, respectively.

The symmetric matrix of weighted connections $W = \{w_{ij}\}, i,j \in \{1,\ldots,K\}$ is simply formed by the absolute value of $r_{i,j} = r_{X_i,X_j} = r_{X,Y}$. For simple connections, the adjacency matrix $A = \{a_{ij}\}, i,j \in \{1,\ldots,K\}$ acquires components assigned the value of 1 when zero cross-correlation is found significant, and zero 0 otherwise. The decision regarding statistical significance is made using the significance test for the correlation coefficient (Horvath 2011). The null hypothesis is $H_0 : \rho_{X,Y} = 0$, where $\rho_{X,Y}$ is the true Pearson's correlation coefficient. The sample cross-correlation coefficient $r_{X,Y}$ is transformed to the test statistic

$$t = r_{X,Y}\sqrt{\frac{n-2}{1-r_{X,Y}^2}}, \tag{2}$$

which follows Student's distribution with $n-2$ degrees of freedom. A connection is significant and equals 1 if the p value of the test is less than a given significance level α (here $\alpha = 0.05$).

Investigation of nontrivial properties, especially the small-world property, and the evolution of network measures, requires the construction of randomized networks. This is a graph that can be created through a random process from an original one. The standard approach is to randomize the connections of the original network while preserving certain characteristics. A simple setting for randomization is the preservation of the total number of connections, or the total strength if the connections are weighted (Newman 2010). A more elaborate randomization setting requires the preservation of the degree of each node in the original network (Molloy and Reed 1995; Maslov and Sneppen 2002; Del Genio et al. 2010), or, respectively, the strength for weighted connections (Opsahl et al. 2008). In a different approach, the network is not formed by randomizing the connections of the original network but is rather built using the Erdős and Rényi (1959) model with the probability of connections as in the original network and a Poissonian degree distribution.

To construct randomized networks when the original network is formed from time series measurements, it is found more appropriate to randomize the original time series rather than the connections of the original network (Chorozoglou and Kugiumtzis 2018). In this scheme, each of the K time series $\{X_{1,t}, \ldots, X_{K,t}\}$ for $t = 1, \ldots, n$ is randomized separately under the condition of preserving the marginal distribution and the autocorrelation function (Kugiumtzis 2002), or equivalently the power spectrum (Schreiber and Schmitz 1996).

2.2. Network Measures

Generally, the connections of networks are undirected or directed and weighted or simple (binary). For any two nodes i and j, the distance between them, denoted as d_{ij} for simple connections and d_{ij}^w for weighted ones, is defined as the length of the shortest path from i to j if the nodes are connected, and $d_{ij} = \infty$ or $d_{ij}^w = \infty$ otherwise. To monitor the earthquake network structure, different network measures are considered and computed, based on either the adjacency matrix A or the weighted matrix W, the mathematical expressions for seven of which that have been used in many studies (Abe and Suzuki 2009; Daskalaki et al. 2016; Chorozoglou et al. 2017) are given in Table 1 and briefly described below:

1. The degree centrality is the average over all nodes of either the number or the strength of connections.
2. The clustering coefficient is the average over all nodes of the fraction of connections between the nodes within their neighborhood, divided by the number of all possible connections.
3. The characteristic path length is the average of the shortest path lengths in the network, computed over all pairs of nodes.
4. The global efficiency is the average inverse shortest path length in the network, computed over all pairs of nodes.
5. The betweenness centrality is the average over all nodes of the node betweenness centrality, which is the fraction of all shortest paths in the network that contain the node, divided by the number of all possible paths.
6. The modularity shows the strength of a network that is divided into components.
7. The eccentricity is the average over all nodes of the maximal shortest path length between a node and any other node.

3. Applications Based on Network Theory

Complex network theory has been used for seismicity investigation, firstly by Abe and Suzuki (2004a) as well as Baiesi and Paczuski (2004, 2005) for Southern California and more recently by Daskalaki et al. (2016) for Italy and Chorozoglou et al. (2017) for Greece. Some of the applications based on

Table 1

Mathematical expressions for seven network measures with simple and weighted undirected connections

Simple connections	Weighted connections
Degree centrality or strength	
$\bar{k} = \sum_{i=1}^{K} k_i, k_i = \sum_{j=1}^{K} a_{ij}$	$\bar{k}^w = \sum_{i=1}^{K} k_i^w, k_i^w = \sum_{j=1}^{K} w_{ij}$
Clustering coefficient	
$C = \frac{1}{K}\sum_{i=1}^{K} c_i = \frac{1}{K}\sum_{i=1}^{K} \frac{\sum_{j,h\in K} a_{ij}a_{ih}a_{jh}}{k_i(k_i-1)}$	$C^W = \frac{1}{K}\sum_{i=1}^{K} c_i^w = \frac{1}{K}\sum_{i=1}^{K} \frac{\sum_{j,h\in K} w_{ij}w_{ih}w_{jh}}{k_i^w\left(k_i^w-1\right)}$
Characteristic path length	
$L = \frac{1}{K}\sum_{i=1}^{K} L_i = \frac{1}{K}\sum_{i=1}^{K} \frac{\sum_{j\in K, j\neq i} d_{ij}}{K-1}$	$L^W = \frac{1}{K}\sum_{i=1}^{K} L_i^w = \frac{1}{K}\sum_{i=1}^{K} \frac{\sum_{j\in K, j\neq i} d_{ij}^w}{K-1}$
Global efficiency	
$E = \frac{1}{K}\sum_{i\in K} E_i = \frac{1}{K}\sum_{i\in K} \frac{\sum_{j\in K, j\neq i} d_{ij}^{-1}}{K-1}$	$E^W = \frac{1}{K}\sum_{i\in K} E_i^W = \frac{1}{K}\sum_{i\in K} \frac{\sum_{j\in K, j\neq i} (d_{ij}^w)^{-1}}{K-1}$
Betweenness centrality	
$\mathrm{b}_i = \frac{1}{(K-1)(K-2)} \sum_{\substack{h,j\in \\ h\neq j, h\neq i, j\neq i}} \frac{\rho_{hj}(i)}{\rho_{hj}}$	$\mathrm{b}_i^w = \frac{1}{(K-1)(K-2)} \sum_{\substack{h,j\in \\ h\neq j, h\neq i, j\neq i}} \frac{\rho_{hj}^w(i)}{\rho_{hj}^w}$
Modularity	
$Q = \sum_{u\in M}\left[e_{uu} - \left(\sum_{v\in M} e_{uv}\right)\right]$	$Q^W = \frac{1}{E^W}\sum_{i,j\in N}\left[w_{ij} - \frac{k_i^w k_j^w}{E^w}\right] \delta_{m_i,m_j}$
Eccentricity	
$e_i = \max_{x\in K}\{d(i,x)\}$	$e_i^W = \max_{x\in K}\{d^W(i,x)\}$

nontrivial properties of networks (Sect. 3.1) and the evolution of network measures (Sect. 3.2) are described below.

3.1. Earthquake Network Properties

The study of complex networks has revealed interesting nontrivial properties that indicate their specific structures, such as the small-world (Watts and Strogatz 1998) and scale-free (Albert and Barabasi 2002) properties, which have been found to indicate underlying organization principles of networks in various scientific fields. The small-world structure is obtained by adding long-range connections to random networks, which increases the effectiveness of information transfer within the network. The small-world network exhibits the characteristic that most nodes can be reached from every other node by a small number of steps, thus the typical distance between two randomly chosen nodes grows proportionally to the number of nodes, K. Therefore, when an earthquake network is characterized as a small-world network, the seismicity between two seismic regions can be correlated through other nodes (seismic regions). The most popular small-world manifestation is the six degrees of separation concept, uncovered by the social psychologist Milgram (1967), which implies long-range connections. The identification of the small-world property in many natural complex networks has stimulated great interest in studying the underlying organizing principles of various complex networks. The small-world property has been identified and studied in diverse scientific fields such as neuroscience (Van den Heuvel et al. 2008; Bialonski et al. 2010; Papo et al. 2016) and meteorology (Hlinka et al. 2012). Abe and Suzuki (2004a, 2006) and Abe et al. (2011) proved the universality of the small-world property for earthquake networks constructed for four different study areas (California, Japan, Iran, and Chile). This property was also studied and verified by Jimenez et al. (2008) for California, Baek et al. (2011) for the Korean Peninsula, Chorozoglou et al. (2018) for Greece, and León et al. (2018) for Colombia. When a scale-free network is recognized, this means that there are a few nodes, i.e., seismic regions, with higher level of seismicity (i.e., hubs) than the others that can influence the other nodes with lower seismicity level. The scale-free property was firstly investigated by Abe and Suzuki (2004b) and Baiesi and Paczuski (2004) for Southern California and more recently by Pastén et al. (2016) for Chile and Janer et al. (2017) for the Philippines and Southern California.

Watts and Strogatz (1998) introduced the term “small world” for networks having a characteristic path length L at the level of the characteristic path length L_{rand} of the randomized network, $\lambda = \frac{L}{L_{\mathrm{rand}}} \approx 1$, but high clustering coefficient C, when compared with the clustering coefficient of a random network C_{rand}, $\gamma = \frac{C}{C_{\mathrm{rand}}} \gg 1$. The small-world property is quantified by the so-called small-worldness index S, defined as

$$S = \frac{\gamma}{\lambda} = \frac{C/C_{\mathrm{rand}}}{L/L_{\mathrm{rand}}} \quad (3)$$

A value of S much larger than 1 implies the existence of the small-world property. Computationally, the network measures C_{rand} and L_{rand} are derived as the average of the clustering coefficients and characteristic path lengths, respectively, from an ensemble of random networks.

A scale-free network (Albert and Barabasi 2002), whose name originates from the power-law distribution of the node degree k_i, includes some nodes that play a central role, since they sustain several connections; i.e., they are hubs. Regardless of the size K of the network, there is nonzero probability of having a node that is connected to almost all others, suggesting that the right tail of the degree distribution does not vanish exponentially but follows a power law. Thus, in scale-free networks, there are many nodes with few connections and few nodes with many connections (Barabasi and Albert 1999). The power-law degree distribution is given by $P(k) \sim k^{-\gamma}$, where γ is the degree exponent, commonly lying in the range $2 \le \gamma \le 3$. Applications in seismology on the nontrivial properties are described below.

3.1.1 Small-World Property

The first study on the small-world property of earthquake networks was performed by Abe and Suzuki (2004a) for California, using data from the Southern California Earthquake Data Center (http://www.scecdc.scec.org/catalogs.html) in the period from 00:40:07 on 1 January to 23:55:34 on 31 December, 1992. The nodes were seismic cells, the connections were defined by succession of earthquakes, and the randomized networks were constructed by using an Erdős and Rényi model. The degrees of separation between two seismic cells, as the nodes that were chosen at random, take a small value between 2 and 3. The clustering coefficient was also calculated and found to be about 10 times larger than in the case of a completely random network, indicating that the small-world structure was dominant in the earthquake network [i.e., $S \gg 1$, see Eq. (3)]. Similarly, Abe and Suzuki (2006) demonstrated the small-world property for California and Japan (Table 2), using data from (i) the Southern California Earthquake Data Center (http://www.data.scec.org/) and (ii) Japan University Network Earthquake Catalog (http://kea.eri.u-tokyo.ac.jp/CATALOG/junec/monthly.html) in the period from (i) 00:25:8.58 on 1 January 1984 to 22:21:52.09 on 31 December 2003 and (ii) 01:14:57.63 on 1 January 1993 to 20:54:38.95 on 31 December 1998, respectively.

The small-world property was then confirmed by Jimenez et al. (2008) for Southern California (Table 3) by the use of time series. The nodes were

Table 2

Summary results for small-world property for California and Japan (Abe and Suzuki 2006)

Cell size	10 km × 10 km × 10 km	5 km × 5 km × 5 km
California	N = 3.869	N = 12.913
	C = 0.630 (C_{rand} = 0.014)	C = 0.317 (C_{rand} = 0.003)
	L = 2.526	L = 2905
Japan	N = 27.599	N = 57.768
	C = 0.045	C = 0.015
	(C_{rand} = 0.000298)	(C_{rand} = 0.0000711)
	L = 3.825	L = 3.923

Table 3

Summary results for small-world property (Jimenez et al. 2008)

Networks	Nodes	Connections	k	L	C
California (10.31 km)	6.945	84.874	12.22	1.26	0.58
California (13.75 km)	4.271	64.185	15.03	1.19	0.64
California (17.18 km)	2.741	50.675	18.89	1.14	0.70

seismic cells, the connections were given by the linear zero-lag cross-correlation [see Eq. (1)], and the randomized networks were constructed by using an Erdős and Rényi (1959) model. Data were taken from the Southern California Earthquake Center (SCEC) covering the period from 1 January 1984 to 3 July 2001. The small-world property means that there are long-range connections in the seismic network, hence, it might be related to the San Andreas Fault or other major faults in the region.

A few years later, Abe et al. (2011) proved the universality of the small-world property for four different study areas (California, Japan, Iran, and Chile). The construction of earthquake networks was similar to the previous study (Abe and Suzuki 2004a); i.e., the nodes were seismic cells, the connections were given by the succession of earthquakes, and the randomized networks were constructed by using an Erdős and Rényi model. Baek et al. (2011) revealed the small-world property for the region of the Korean Peninsula, using data from August 1978 to January 2010 and an earthquake network construction approach according to Abe and Suzuki (2004a).

Chorozoglou et al. (2018) revealed the small-world property, using time series, for the region of Greece before nine main shocks during the period from 1999 to 2015 (Fig. 2). The nodes were seismic regions, the connections were given by the test significance of the linear zero-lag cross-correlation [see Eq. (2)], and the randomized networks were constructed using the approach of Chorozoglou and Kugiumtzis (2018). The seismic catalog compiled from the Geophysics Department of the Aristotle University of Thessaloniki (http://geophysics.geo.auth.gr/ss/) was the data source. Complex network theory revealed that the topological measure, viz. the small-world index, could serve as a potential index for short-term seismic hazard assessment, as the index S increased rapidly from the level of one ($S \gg 1$) before the occurrence of main shocks (Fig. 2). León et al. (2018) revealed the small-world property for the region of Colombia (Table 4) using

Table 4

Summary results for small-world property for Colombia (León et al. 2018)

km	C	C_{rand}	L	L_{rand}	S
10	0.0105	0.0012	4.347	6.852	13.7923
20	0.0164	0.0027	3.273	4.940	9.1670
30	0.027	0.0055	2.815	4.003	6.9808
40	0.0529	0.0105	2.649	3.367	6.4036
50	0.0825	0.018	2.48	2.97	5.4089
60	0.1021	0.026	2.342	2.727	4.5725
70	0.1334	0.0385	2.21	2.521	3.9525
80	0.1711	0.054	2.123	2.35	3.5073
90	0.2349	0.076	2.075	2.147	3.1960
100	0.3027	0.105	2.034	1.98	2.8063

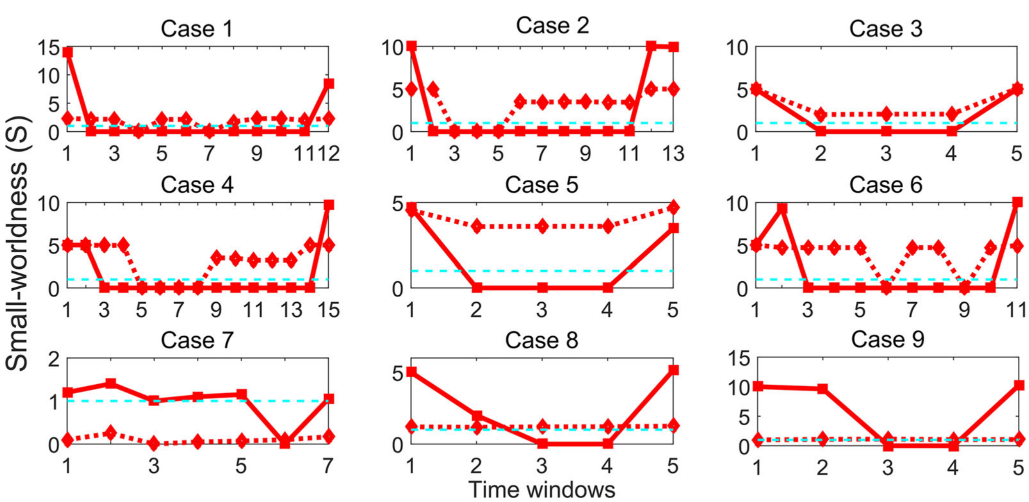

Figure 2

Evolution of the small-worldness index S for earthquake networks using two approaches for random network generation (solid and dashed line for RTSbinthr and RTSweight, respectively). The number of windows on the x-axis varies across the different records of intershock seismic activity (cases 1–9) because the intershock intervals are different (Chorozoglou et al. 2018)

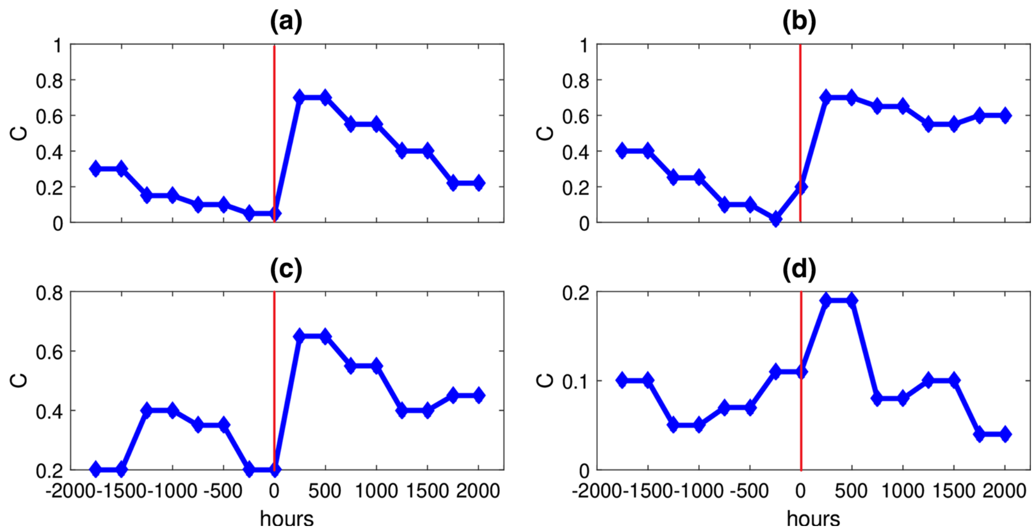

Figure 3
Evolution of clustering coefficient for four strong earthquakes (Abe and Suzuki 2009)

data taken from the National Earthquake Information Center, retrieving the list of earthquakes for the Colombian region. The earthquake network construction was based on Abe and Suzuki (2004a).

3.1.2 Scale-Free Property

Abe and Suzuki (2004b) first revealed the scale-free property for Southern California, using data from the Southern California Earthquake Data Center (http://www.scecdc.scec.org/catalogs.html) for the time interval between 00:40:07.47 on 1 January 1992 and 23:55:34.66 on 31 December 1992. The data well obey the power-law distribution, which may be interpreted as follows: The Gutenberg–Richter law, on the one hand, tells us that the frequency of earthquakes with large values of moment decays as a power law (i.e., heavy-tail behavior), showing the existence of nonnegligible numbers of strong earthquakes. On the other hand, aftershocks associated

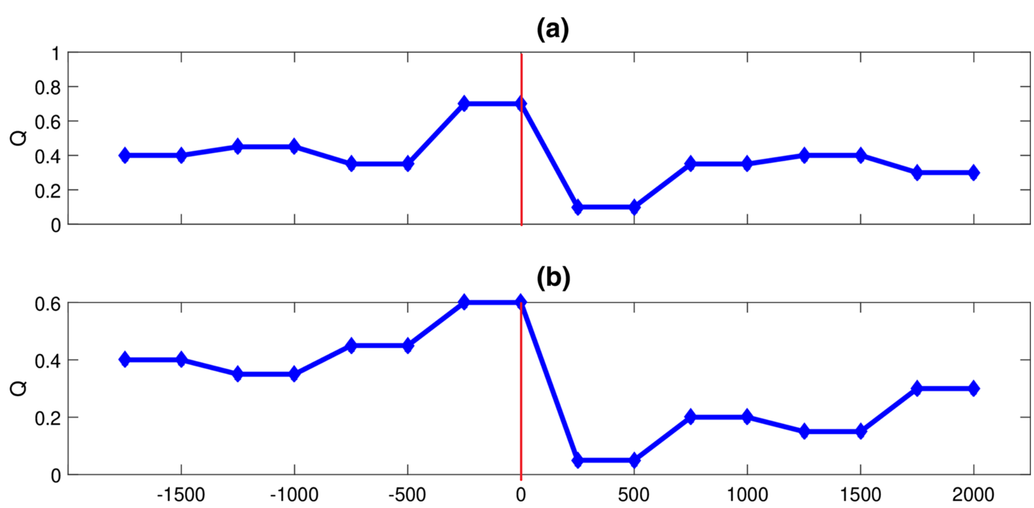

Figure 4
Evolution of maximum value of modularity measure, Q_{max}, around (1) the Joshua Tree Earthquake, (2) the Landers Earthquake, and (3) the Hector Mine Earthquake. The moments of the main shocks are located at the origin. In each case, the values of the cell size are: **a** 5 km × 5 km × 5 km and **b** 10 km × 10 km × 10 km (Abe and Suzuki 2006)

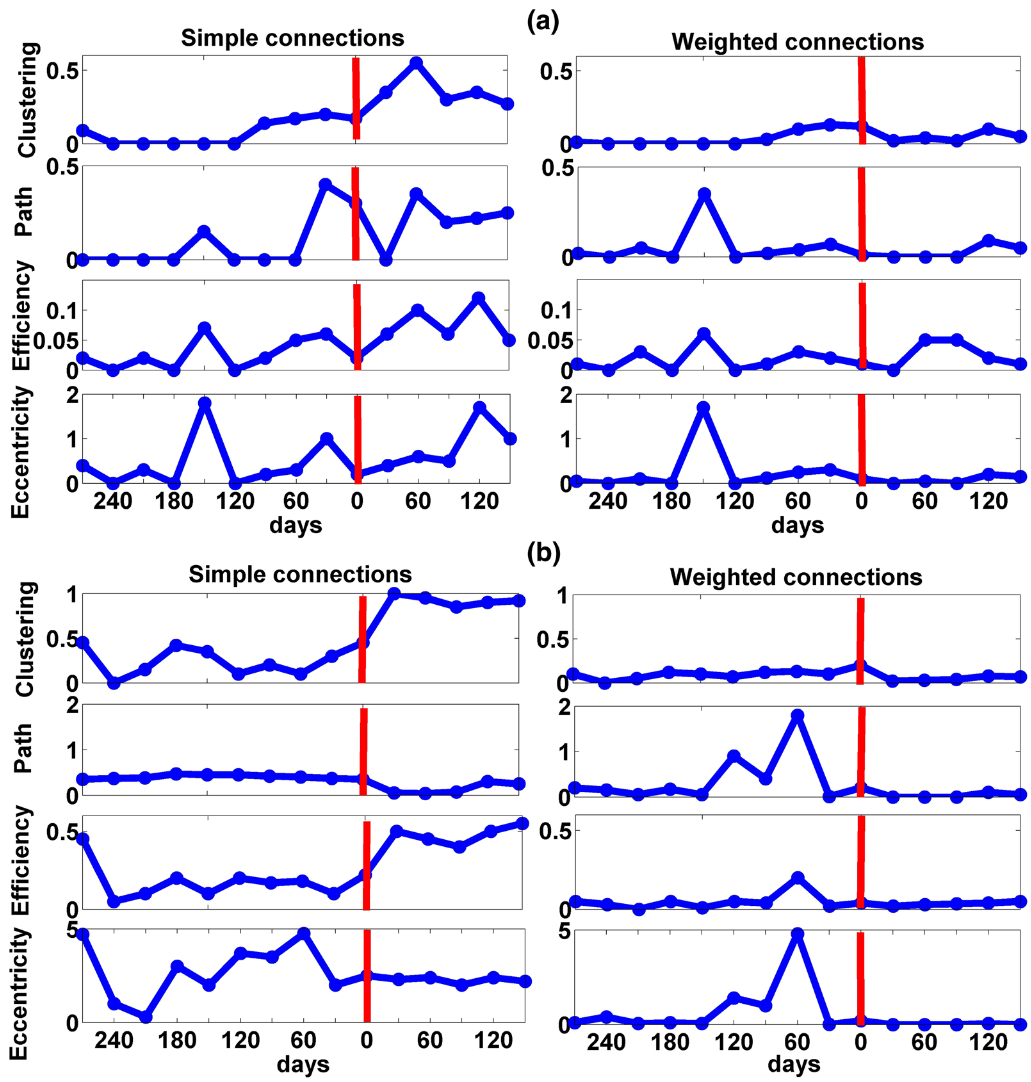

Figure 5
Evolution of values of network measures for the main shock **a** near Kefalonia and **b** near Methoni, where the red vertical line denotes the time of main shock. The seismic measure for introducing the weighted connections is the number of earthquakes (Chorozoglou et al. 2017)

with a main shock tend to be connected to the node of the main shock. Thus, the scale-free nature of the connectivity distribution is consistent with the Gutenberg–Richter law.

In addition, Baiesi and Paczuski (2004) revealed the scale-free property for Southern California, using data from the Southern California Earthquake Data Center (http://www.data.scec.org/ftp/catalogs/SCSN) from 1 January 1984 to 31 December 2003, and network construction based on Bak et al. (2002). The scale-free property was also studied by Baek et al. (2011) for the Korean Peninsula and by Pastén et al. (2016) for Chile. The seismic data for the Korean Peninsula were collected from August 1978 to January 2010, and for Chile from 04:21:57.0 on 2 October 2000 to 18:31:57.3 on 29 March 2007.

3.2. Evolution of Network Measures

The evolution of some earthquake network measures exhibits variations that may serve as potential indices for short-term seismic hazard assessment.

Figure 3 presents the evolution of the clustering coefficient before and after four main shocks (Abe and Suzuki 2009). For the four cases, the values of the clustering coefficient reveal an abrupt jump shortly before the occurrence of the main shock. To ascertain the universality of this finding, several strong shocks that occurred in different geographical regions are considered, namely the Joshua Tree Earthquake ($M = 6.1$, 23 April 1992), the Landers Earthquake ($M = 7.3$, 28 June 1992), the Hector Mine Earthquake ($M = 7.1$, 16 October 1999) in California (database: http://www.data.scec.org), and the Kushiro-Oki Earthquake ($M = 7.1$, 29 November 2004) in Japan. The same analysis as in the previous work but with the modularity as a network measure (Fig. 4) was applied by Abe and Suzuki (2006) and for the betweenness centrality by Daskalaki et al. (2016).

Chorozoglou et al. (2017) focused on the cases of the $M = 6.1$ main shock that occurred in the Paliki Peninsula of Kefalonia Island on 26 January 2014, and the $M = 6.8$ main shock that occurred offshore southwestern Peloponnese, on 14 February 2008. The seismic catalog compiled by the Geophysics Department of the Aristotle University of Thessaloniki (http://geophysics.geo.auth.gr/ss/) was the data source, and the evolution of the values of some network measures before and after the two main shocks was investigated (Fig. 5). The network measures tended to be stable with small variations before both main shocks, then exhibited an abrupt jump shortly before their occurrence, and finally slowly decreased and became stable again.

4. Conclusions

The network approach has been found to be a powerful tool that can contribute significantly to the investigation of properties of complex phenomena such as seismic activity. The results of applications, presented here, suggest the effectiveness of network analysis for seismicity studies in earthquake-prone areas. The study of complex earthquake networks has revealed nontrivial properties that indicate their specific structures, i.e., the small-world and scale-free properties, which have been found to indicate the underlying organization principles of earthquake networks. Hence, the universality of the small-world property for four different study areas (California, Japan, Iran, and Chile) was proved by Abe et al. (2011), for the Korean Peninsula by Baek et al. (2011), and for Colombia by León et al. (2018).

Statistically significant changes in the network structure are shown by certain global measures, being observed simultaneously a few days before main shocks. Network measures can track the changes in the structure evolution of earthquake networks and can be regarded as proxies of the seismicity behavior. The key topological measures, such as the clustering coefficient (Abe and Suzuki 2009), modularity (Abe and Suzuki 2006), betweenness centrality (Daskalaki et al. 2016), and small-world property (Chorozoglou et al. 2018), may serve as potential indices for short-term seismic hazard assessment.

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