Application of Superstatistics in Hellenic Seismicity

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ABSTRACT

The complex turbulent dynamics of seismogenesis in the area of Greece is investigated, applying the recently developed superstatistics theory [1] to three earthquake interevent time series. The corresponding three data sets are subsets of the regional earthquake catalogue compiled in the Seismological Station of Geophysics Department, Aristotle University of Thessaloniki and concern: 1221 earthquakes with M≥5.2, occurred during 1911–2017; 576 earthquakes with M≥5.5, during 1911–2017 and 114 earthquakes with M≥6.5, in the period of 1845–2017. In the superstatistics' framework, the Hellenic seismogenesis can be seen as a driven nonequilibrium system that is spatially consisted of smaller subregions (e.g. earthquake zones) exhibiting spatio-temporal fluctuations of an intensive quantity *β* (e.g. stress field). Then, its complex behavior can be efficiently described by a superposition of different dynamics on variable time scales and in particular two considerably distinctive time scales, *τ* and *T*, corresponding to fast (relaxation time to local equilibrium) and slow dynamics (the long time scale in which the intensive quantity *β* fluctuates). In order to test this hypothesis, we adopt the methodology described in [2] investigating whether this time scale separation exists, as well as if the slowly varying stochastic process *β(t)* falls into the category of the three major universal classes of superstatistics, namely χ^2 (related to Tsallis statistics), inverse χ^2 and log-normal superstatistics. In particular, we estimate the superstatistical time scales *τ* (exponential decay of autocorrelation functions) and *T* (the value for which the flatness coefficient equals to 3) and their ratio *T/τ*. Then, we evaluate the probability density of the superstatistical process *β*(t) and compare it with χ^2, inverse χ^2 and log-normal distributions generated with the same length, mean and variance as *β*(t).

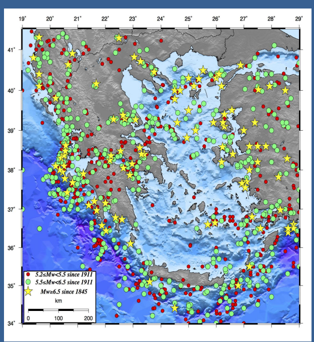

Figure 1. Epicentral distribution of earthquakes considered in this study.

METHODS AND MATERIALS

1. Determination of Large time scale *T*
   - Construct differenced series, *u(t)=u(t+δ)-u(t)*, $\delta=2^j$, *j=0,1,2,…,n*.
   - Estimation of Flatness Coefficient:

$$F(\Delta t)=\frac{1}{t_{\max}-\Delta t}\int_0^{t_{\max}-\Delta t} dt_0 \frac{\left\langle (u-\bar{u})^4 \right\rangle_{t_0,\Delta t}}{\left\langle (u-\bar{u})^2 \right\rangle_{t_0,\Delta t}^2}$$

   - Define the superstatistical time scale *T* by the condition *F(T)=3*.
2. Determination of short time scale τ from exponential decay of autocorrelation coefficients estimated for the differenced series *u(t)*.
3. Estimation of the time scale ratio *T/τ*. If *T/τ*>>1 then there exists a clear separation of time scales, namely fast and slow.
4. Generation of slowly varying stochastic process *β(t)* = 1/ variance(*u(t)*).
5. Comparison of *β*(*t*) PDF with χ^2, inverse χ^2 and log-normal distributions with the same length, mean and variance with *β*(*t*).

REFERENCES

1. C. Beck and E. G. D. Cohen, Superstatistics, *Physica A 322, (2003), 26.*
2. C. Beck, E.G.D. Cohen, H.L. Swinney, From time series to superstatistics, *Physical Review E 72, (2005) 056133.*
3. A.C. Iliopoulos, G.P. Pavlos, E.E. Papadimitriou, D.S. Sfiris, Chaos, self organized criticality, intermittent turbulence and nonextensivity revealed from seismogenesis in north Aegean area, *International Journal of Bifurcation and Chaos 22(9) (2012), 1250224.*
4. A.C. Iliopoulos, G.P. Pavlos, Global low dimensional chaos in the Hellenic region, *International Journal of Bifurcation and Chaos 20(7) (2010), 2071-2095.*

RESULTS

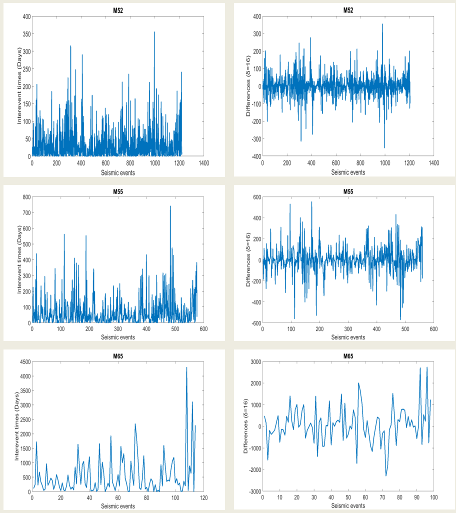

Figure 2. (left column) Earthquake interevent time series with magnitudes M≥5.2, M≥5.5 and M≥6.5, respectively. **(right column)** An example of differenced series *u*(*t*) generated with *δ*=16 for the interevent time series shown in left column.

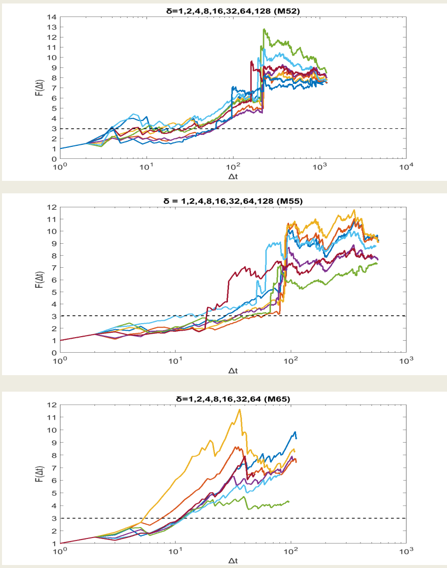

Figure 3. Flatness coefficient *F(Δt)* as a function of Δt, estimated for all differenced series *u*(*t*) generated with δ=1,2,4,8,16,32,64,128, for the three interevent times, respectively.

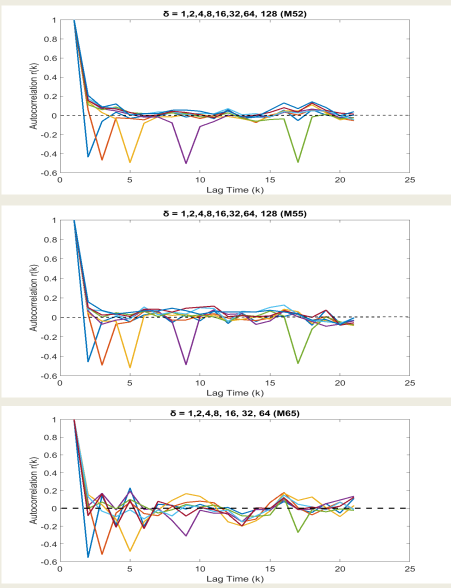

Figure 4. Autocorrelation coefficients *r*(*k*) estimated as a function of lag time *k* estimated for the differenced series *u*(*t*) (*δ*=1,2,4,8,16,32,64,128), for each of the interevent time series, respectively.

RESULTS

Table 1. Values of long time scale *T* estimated for Gaussian simple superstatistics, namely *F*(*T*)=3 for each differenced series *u*(*t*) of the three interevent time series.

δ	*T* (M52)	δ	*T* (M55)	δ	*T* (M65)
1	24.02	1	26.89	1	11.18
2	62.80	2	53.03	2	7.66
4	8.04	4	33.93	4	5.65
8	62.64	8	33.28	8	11.20
16	35.01	16	41.80	16	11.54
32	19.33	32	11.89	32	11.84
64	6.94	64	18.01	64	10.99
128	63.77	128	48.64	-	-

Table 2. Values of short time scale *T* estimated from the exponential decay of the autocorrelation coefficients for each differenced series *u*(*t*) of the three interevent time series.

δ	τ (M52)	δ	τ (M55)	δ	τ (M65)
1	1.41	1	1.42	1	1.40
2	1.30	2	1.33	2	1.32
4	1.14	4	1.19	4	1.13
8	1.2	8	1.30	8	1.30
16	1.26	16	1.28	16	1.31
32	1.22	32	1.29	32	1.20
64	1.23	64	1.28	64	1.21
128	1.21	128	1.26	-	-

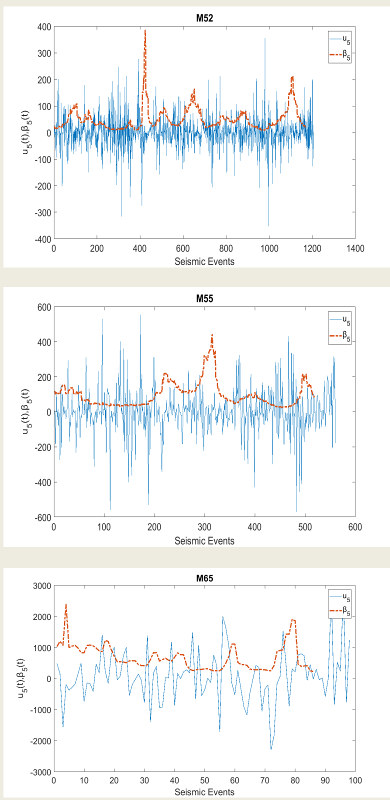

Figure 5. Time series of the process *β*(*t*) (top-orange) and differenced series *u*(*t*). For these examples *δ*=5 and *T* = 35, 42, 12 respectively. As it can be seen *β*(*t*) is a slow varying process.

RESULTS

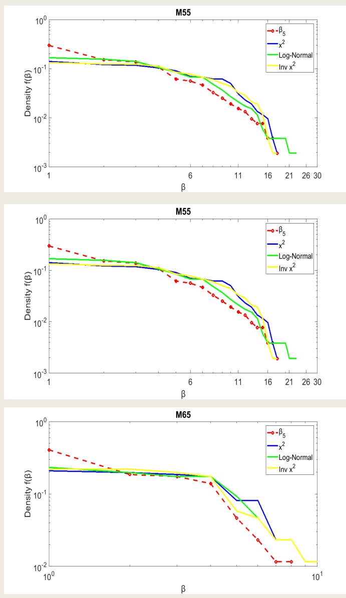

Figure 4. Probability density *f(β) extracted* from the time series *θ*(*t*) (*δ*=16), and compared with log-normal, χ^2, and inverse χ^2 distributions on log-log plots. The log-normal, χ^2, and inverse χ^2 distributions were generated using the same mean and variance as *f*(*β*).

Table 3. Time scale ratio *T/τ*, based on results presented on Tables 1 and 2. All vallues are much greater than unity, indicating a clear separation of time scales.

δ	M52	M55	M65
1	17.03	18.93	7.98
2	48.30	39.87	5.80
4	7.05	28.51	5.00
8	52.20	25.60	8.61
16	27.78	32.65	8.81
32	15.84	9.21	9.86
64	5.64	14.07	9.08
128	52.70	38.60	-

CONCLUSIONS

The results reveal :

- Fat-tailed non-Gaussian distributions for interevent time series, since F > 3 in all three cases.
- The presence of two separate time scales, verifying superstatistics theory prediction, since the time scale ratio *T/τ* attains values much higher from unity in all cases.
- Log-normal superstatistics fit better the intensive parameter *β* for earthquake interevent times M52 and M55, while for M65 no safe conclusions can be drawn. This result means that M52 and M55 interevent time series can be described by local Boltzmann factors $e^{-(\beta u^2)/2}$, whose variance parameter *β* varies slowly according to a log-normal distribution function.
- These results are also related to previous studies [3, 4] evidencing the non-extensive chaotic character of the Hellenic seismogenesis.

Work Perspective: The used time series are relatively short and therefore additional analysis is required to verify the aforementioned findings.

ACKNOWLEDGMENTS

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